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n-ARY 2-ABSORBING AND 2-ABSORBING PRIMARY HYPERIDEALS IN KRASNER $(\mathbf{m},\mathbf{n})$-HYPERRINGS

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Abstract. Let R be a commutative Krasner (m, n)-hyperring with the scalar identity 1_R. In this paper, we introduce and study the concept of n-ary 2-absorbing and 2-absorbing primary hyperideals of R. These concepts are a generalisation of n-ary prime and primary hyperideals.

1. Introduction

The theory of hyperstructures is a well established branch in classical algebraic theory. Since 1934, when Marty [14] introduced for the first time the notion of a hypergroup, the Hyperstructure Theory has had applications to several domains, for instance graphs and hypergraphs, non-Euclidean geometry, lattices, binary relations, cryptography, automata, artificial intelligence, codes, probabilities etc (see [5–7, 18]). Recently, Davvaz and Vougiouklis have introduced and studied a nice generalization of a hypergroup which is called an n-hypergroup [8].

n-ary semigroups and n-ary groups are algebras with one n-ary operation which is associative and invertible in a generalized sense. The investigations of n-ary algebras go back to Krasner's lecture [11] at the 53rd annual meeting of the American Association of the Advancement of Science in 1904. But the first paper concerning the theory of n-ary groups was written by Dörente in 1928 [9]. Afterward, the (m, n)-rings and their quotient structure were introduced by Crombez and Timm in [3,4]. The concept of an n-ary hypergroup was defined by Davvaz and Vougiouklis in [8], which is a generalization of the concept of a hypergroup in the sense of Marty and a generalization of an n-ary group, too. The notation of (m, n)-hyperrings was defined by Mirvakili and Davvaz [15] and they obtained (m, n)-rings from (m, n)-hyperrings by using fundamental relations. For more study on n-ary structures and n-ary hyperstructures refer to [12, 13, 16].

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The concept of 2-absorbing ideals, in ordinary algebra, was introduced by A. Badawi, in [2]. In 2017, Davvaz et al. introduced the concept of 2-absorbing fuzzy ideals and 2-absorbing primary fuzzy ideals in commutative rings [17]. In [10], the notion of (k, n)-absorbing hyperideals was studied in Krasner (m, n)-hyperirings by Hila et al.

In this paper, we aim to introduce and study the notion of n-ary 2-absorbing and n-ary 2-absorbing primary hyperideals in Krasner (m, n)-hyperrings. The concept is a generalisation of n-ary prime and primary hyperideals which were studied by R. Ameri in [1].

Among the results in this paper, it is shown (Theorem 3.6) that there are at most two n-ary prime hyperideals of (R, f, g) that are minimal over an n-ary 2-absorbing hyperideal I of R. It is shown (Theorem 3.8) that if I is an n-ary primary hyperideal of a commutative Krasner (m, n)-hyperring (R, f, g) with the scalar identity 1_R such that $\sqrt{I}^{(m,n)} = P$ for some n-ary prime hyperideal P of R, then I is an n-ary 2-absorbing hyperideal of R if and only if $g(P^{(2)}, 1_R^{(n-2)}) \subseteq I$. In Section 4, we investigate the stability of n-ary 2-absorbing hyperideals in some hyperring-theoretic constructions. In Section 5, we introduce and study the concept of n-ary 2-absorbing primary hyperideals.

2. Preliminaries

In this section we recall some definitions and results concerning n-ary hyperstructures which we will use later.

A mapping $f : H^n \longrightarrow P^*(H)$ is called an *n-ary hyperoperation*, where $P^*(H)$ is the set of all the non-empty subsets of H. An algebraic system (H, f), where f is an n-ary hyperoperation defined on H, is called an *n-ary hypergroupoid*.

We shall use the following abbreviated notation: The sequence $x_i, x_{i+1}, \ldots, x_j$ will be denoted by x_i^j. For $j \prec i$, x_i^j is the empty symbol. With this convention $f(x_1, \ldots, x_i, y_{i+1}, \ldots, y_j, z_{j+1}, \ldots, z_n)$ will be written as $f(x_1^i, y_{i+1}^j, z_{j+1}^n)$. In the case when $y_{i+1} = \ldots = y_j = y$ the last expression will be written in the form $f(x_1^i, y^{(j-i)}, z_{j+1}^n)$.

For non-empty subsets $A_1, \ldots, A_n$ of H we define $f(A_1^n) = f(A_1, \ldots, A_n) = \bigcup\{f(x_1^n) \mid x_i \in A_i, i = 1, \ldots, n\}$. An n-ary hyperoperation f is called *associative* if $f(x_1^{i-1}, f(x_i^{n+i-1}), x_{n+i}^{2n-1}) = f(x_1^{j-1}, f(x_j^{n+j-1}), x_{n+j}^{2n-1})$ holds for every $1 \leq i \prec j \leq n$ and all $x_1, x_2, \ldots, x_{2n-1} \in H$. An n-ary hypergroupoid with the associative n-ary hyperoperation is called an *n-ary semihypergroup*.

An n-ary hypergroupoid (H, f) in which the equation $b \in f(a_1^{i-1}, x_i, a_{i+1}^n)$ has a solution $x_i \in H$ for every $a_1^{i-1}, a_{i+1}^n, b \in H$ and $1 \leq i \leq n$ is called an *n-ary quasihypergroup*. When (H, f) is an n-ary semihypergroup, (H, f) is called an *n-ary hypergroup*.

An n-ary hypergroupoid (H, f) is *commutative* if for all $\sigma \in \mathbb{S}_n$, the group of all permutations of $\{1, 2, 3, \ldots, n\}$, and for every $a_1^n \in H$ we have $f(a_1, \ldots, a_n) =$

$f(a_{\sigma(1)}, \ldots, a_{\sigma(n)})$. If an $a_1^n \in H$ we denote $a_{\sigma(1)}^{\sigma(n)}$ as the $(a_{\sigma(1)}, \ldots, a_{\sigma(n)})$. We assume throughout this paper that all Krasner (m, n)-hyperrings are commutative.

If f is an n-ary hyperoperation and $t = l(n-1)+1$, then t-ary hyperoperation $f_{(l)}$ is given by $f_{(l)}(x_1^{l(n-1)+1}) = f(f(\ldots, f(f(x_1^n), x_{n+1}^{2n-1}), \ldots), x_{(l-1)(n-1)+1}^{l(n-1)+1})$.

Definition 2.1 ([15]). Let (H, f) be an n-ary hypergroup and B be a non-empty subset of H. B is called an *n-ary subhypergroup* of (H, f) if $f(x_1^n) \subseteq B$ for $x_1^n \in B$, and the equation $b \in f(b_1^{i-1}, x_i, b_{i+1}^n)$ has a solution $x_i \in B$ for every $b_1^{i-1}, b_{i+1}^n, b \in B$ and $1 \leq i \leq n$. An element $e \in H$ is called a *scalar neutral element* if $x = f(e^{(i-1)}, x, e^{(n-i)})$, for every $1 \leq i \leq n$ and for every $x \in H$.

An element 0 of an n-ary semihypergroup (H, g) is called a *zero element* if for every $x_2^n \in H$ we have $g(0, x_2^n) = g(x_2, 0, x_3^n) = \ldots = g(x_2^n, 0) = 0$. If 0 and $0'$ are two zero elements, then $0 = g(0', 0^{(n-1)}) = 0'$ and so the zero element is unique.

Definition 2.2 ([12]). Let (H, f) be a n-ary hypergroup. (H, f) is called a *canonical n-ary hypergroup* if
(i) there exists a unique $e \in H$, such that for every $x \in H, f(x, e^{(n-1)}) = x$;

(ii) for all $x \in H$ there exists a unique $x^{-1} \in H$, such that $e \in f(x, x^{-1}, e^{(n-2)})$;

(iii) if $x \in f(x_1^n)$, then for all i, we have $x_i \in f(x, x^{-1}, \ldots, x_{i-1}^{-1}, x_{i+1}^{-1}, \ldots, x_n^{-1})$.
We say that e is the scalar identity of (H, f) and x^{-1} is the inverse of x. Notice that the inverse of e is e.

Definition 2.3 ([15]). A *Krasner (m, n)-hyperring* is an algebraic hyperstructure (R, f, g) which satisfies the following axioms:
(i) (R, f) is a canonical m-ary hypergroup;

(ii) (R, g) is a n-ary semigroup;

(iii) the n-ary operation g is distributive with respect to the m-ary hyperoperation f, i.e., for every $a_1^{i-1}, a_{i+1}^n, x_1^m \in R$, and $1 \leq i \leq n$, $g(a_1^{i-1}, f(x_1^m), a_{i+1}^n) = f(g(a_1^{i-1}, x_1, a_{i+1}^n), \ldots, g(a_1^{i-1}, x_m, a_{i+1}^n))$;

(iv) 0 is a zero element (absorbing element) of the n-ary operation g, i.e., for every $x_2^n \in R$ we have $g(0, x_2^n) = g(x_2, 0, x_3^n) = \ldots = g(x_2^n, 0) = 0$.

A non-empty subset S of R is called a *subhyperring of* R if (S, f, g) is a Krasner (m, n)-hyperring. Let I be a non-empty subset of R, we say that I is a hyperideal of (R, f, g) if (I, f) is an m-ary subhypergroup of (R, f) and $g(x_1^{i-1}, I, x_{i+1}^n) \subseteq I$, for every $x_1^n \in R$ and $1 \leq i \leq n$.

Definition 2.4 ([1]). A hyperideal P of a Krasner (m, n)-hyperring (R, f, g), such that $P \neq R$, is called an *n-ary prime hyperideal* if for hyperideals $U_1, \ldots, U_n$ of R, $g(U_1^n) \subseteq P$ implies that $U_1 \subseteq P$ or $U_2 \subseteq P$ or ... or $U_n \subseteq P$.

Lemma 2.5 ([1, Lemma 4.5]). *Let $P \neq R$ be a hyperideal of a Krasner (m, n)-hyperring (R, f, g). Then P is an n-ary prime hyperideal if for all $x_1^n \in R$, $g(x_1^n) \in P \Longrightarrow x_1 \in P$ or ... or $x_n \in P$.*

Definition 2.6 ([1]). Let I be a hyperideal in a (m, n)-hyperring (R, f, g) with scalar identity. The radical (or nilradical) of I, denoted by $\sqrt{I}^{(m,n)}$ is the hyperideal $\bigcap P$, where the intersection is taken over all n-ary prime hyperideals P which contain I. If the set of all n-ary hyperideals containing I is empty, then $\sqrt{I}^{(m,n)}$ is defined to be R.

Ameri and Norouzi [1] showed that if $x \in \sqrt{I}^{(m,n)}$ then there exists $t \in \mathbb{N}$ such that $g(x^{(t)}, 1_R^{(n-t)}) \in I$ for $t \leq n$, or $g_{(l)}(x^{(t)}) \in I$ for $t = l(n-1)+1$.

Definition 2.7 ([1]). A hyperideal $Q \neq R$ in a Krasner (m, n)-hyperring (R, f, g) with the scalar identity 1_R is said to be *n-ary primary* if $g(x_1^n) \in Q$ and $x_i \notin Q$ implies that $g(x_1^{i-1}, 1_R, x_{i+1}^n) \in \sqrt{Q}^{(m,n)}$.

If Q is an n-ary primary hyperideal in a Krasner (m, n)-hyperring (R, f, g) with the scalar identity 1_R, then $\sqrt{Q}^{(m,n)}$ is n-ary prime. (see [1, Theorem 4.28])

Definition 2.8 ([1]). Let S be a hyperideal of a Krasner (m, n)-hyperring (R, f, g). Then the set $R/S = \{f(x_1^{i-1}, S, x_{i+1}^m) \mid x_1^{i-1}, x_{i+1}^m \in R\}$ endowed with m-ary hyperoperation f such that for all $x_{11}^{1m}, \ldots, x_{m1}^{mm} \in R$,

$$\begin{aligned}&f(f(x_{11}^{1(i-1)}, S, x_{1(i+1)}^{1m}), \ldots, f(x_{m1}^{m(i-1)}, S, x_{m(i+1)}^{mm}))\\ =&f(f(x_{11}^{m1}), \ldots, f(x_{1(i-1)}^{m(i-1)}), S, f(x_{1(i+1)}^{m(i+1)}), \ldots, f(x_{1m}^{mm}))\end{aligned}$$

and with n-ary hyperoperation g such that for all $x_{11}^{1m}, \ldots, x_{n1}^{nm} \in R$,

$$\begin{aligned}&g(f(x_{11}^{1(i-1)}, S, x_{1(i+1)}^{1m}), \ldots, f(x_{n1}^{n(i-1)}, S, x_{n(i+1)}^{nm}))\\ =&f(g(x_{11}^{n1}), \ldots, g(x_{1(i-1)}^{n(i-1)}), S, g(x_{1(i+1)}^{n(i+1)}), \ldots, f(x_{1m}^{nm}))\end{aligned}$$

construct a Krasner (m, n)-hyperring, and $(R/S, f, g)$ is called the quotient Krasner (m, n)-hyperring of R by S.

Definition 2.9 ([15]). Let (R_1, f_1, g_1) and (R_2, f_2, g_2) be two Krasner (m, n)-hyperrings. A mapping $\phi : R_1 \longrightarrow R_2$ is called a homomorphism if for all $x_1^m \in R_1$ and $y_1^n \in R_1$ we have $\phi(f_1(x_1, \ldots, x_m)) = f_2(\phi(x_1), \ldots, \phi(x_m))$ $\phi(g_1(y_1, \ldots, y_n)) = g_2(\phi(y_1), \ldots, \phi(y_n))$.

3. n-ary 2-absorbing hyperideals in a Krasner (m, n)-hyperring

Definition 3.1. A nonzero proper hyperideal I of a Krasner (m, n)-hyperring (R, f, g) with the scalar identity 1_R is said to be *n-ary 2-absorbing* if for $x_1^n \in R$, $g(x_1^n) \in I$ implies that $g(x_i, x_j, 1_R^{(n-2)}) \in I$ for some $1 \leq i \prec j \leq n$.

Example 3.2. Let $(R, +, .)$ be a Krasner hyperring in which the operation ".“ is the ordinary multiplication and let R be a hyperintegral domain (for more details refer to [19]). Then R endowed with the following m-ary hyperoperation f and n-ary operation g is a Krasner (m,n)-hyperring: $f(x_1^m)=\sum_{i=1}^m x_i$ and $g(x_1^n)=x_1\dots x_n$. In the Krasner (m,n)-hyperring, the hyperideal $\{0\}$ is an n-ary 2-absorbing hyperideal.

By [1, Example 4.2], the Krasner (m,n)-hyperring R is an n-ary hyperintegral domain. Thus if $g(x_1^n)\in\{0\}$ for some $x_1^n\in R$, then there exist i, $1\le i\le n$ such that $x_i=0$. Hence for all $1\le j\le n$ such that $i\neq j$, we have $g(0,x_j,1_R^{(n-2)})=0$. Therefore $\{0\}$ is an n-ary 2-absorbing hyperideal.

Theorem 3.3. *Let P_1 and P_2 be two n-ary prime hyperideals of a Krasner (m,n)-hyperring (R,f,g) with the scalar identity 1_R. Then $P_1\cap P_2$ is an n-ary 2-absorbing hyperideal of R.*

Proof. Assume that $x_1^n\in R$ such that $g(x_1^n)\in P_1\cap P_2$. If $x_i\in P_1\cap P_2$ for some $1\le i\le n$, then $g(x_i,x_j,1_R^{(n-2)})\in P_1\cap P_2$ for every j, $1\le j\le n$ such that $i\neq j$. Thus we are done. Since P_1 is an n-ary prime hyperideal of R and $g(x_1^n)\in P_1$, we conclude that $x_1\in P_1$ or $\dots$or $x_n\in P_1$. Without losing the generality, we may assume that $x_i\in P_1$ and $x_i\notin P_2$ for some $1\le i\le n$. Since P_2 is an n-ary prime hyperideal of R, we have $x_1\in P_2$ or $\dots$or $x_{i-1}\in P_2$ or $x_{i-1}\in P_2$ or $\dots$or $x_n\in P_2$. Without losing the generality, we may assume that $x_j\in P_2$ such that $i\neq j$. Thus $g(x_i,x_j,1_R^{(n-2)})\in P_1\cap P_2$. Hence $P_1\cap P_2$ is an n-ary 2-absorbing hyperideal of R. $\square$

Theorem 3.4. *Suppose that I is an n-ary 2-absorbing hyperideal of a Krasner (m,n)-hyperring (R,f,g) with the scalar identity 1_R. Then $\sqrt{I}^{(m,n)}$ is an n-ary 2-absorbing hyperideal of (R,f,g) and $g(x^{(2)},1_R^{(n-2)})\in I$ for every $x\in\sqrt{I}^{(m,n)}$.*

Proof. Let I be an n-ary 2-absorbing hyperideal of (R,f,g) and $x\in\sqrt{I}^{(m,n)}$. Then there exists $t\in\mathbb{N}$ such that $g(x^{(t)},1_R^{(n-t)})\in I$ for $t\le n$, or $g_{(l)}(x^{(t)})\in I$ for $t=l(n-1)+1$. If $g(x^{(t)},1_R^{(n-t)})\in I$ for $t\le n$, then

$$\begin{aligned}
&g(g(x^{(t)},1_R^{(n-t)}),1_R^{(n-1)})\in I\\
&\Rightarrow g(x^{(2)},g(x^{(t-2)},1_R^{(n-t+2)}),1_R^{(n-3)})\in I \qquad \text{(associativity)}\\
&\Rightarrow g(x^{(2)},1_R^{(n-2)})\in I \text{ or } g(x,g(x^{(t-2)},1_R^{(n-t+2)}),1_R^{(n-2)})\in I \qquad (I\ n\text{-ary 2-absorbing})\\
&\Rightarrow g(x^{(2)},1_R^{(n-2)})\in I \text{ or } g(x^{(2)},g(x^{(t-3)},1_R^{(n-t+3)}),1_R^{(n-3)})\in I\\
&\Rightarrow g(x^{(2)},1_R^{(n-2)})\in I \text{ or } g(x^{(2)},1_R^{(n-2)})\in I \text{ or } g(x,g(x^{(t-3)},1_R^{(n-t+3)}),1_R^{(n-2)})\in I\\
&\vdots\\
&\Rightarrow g(x^{(2)},1_R^{(n-2)})\in I \text{ or } g(x^{(2)},1_R^{(n-2)})\in I \text{ or } \dots \text{ or } g(x^{(2)},1_R^{(n-2)})\in I.
\end{aligned}$$

If $g_{(l)}(x^{(t)})\in I$ for $t=l(n-1)+1$, then the claim follows by using a similar argument to the previous part and [1, Lemma 4.26]. $\square$

Lemma 3.5. *Let $I\subseteq P$ be a hyperideal of a Krasner (m,n)-hyperring (R,f,g) with the scalar identity 1_R, where P is an n-ary prime hyperideal. Then the following conditions are equivalent:*

(i) P is a minimal n-ary prime hyperideal of I.

(ii) For each $x \in P$, there is a $y \in R \setminus P$ and a nonnegative integer t such that $g(x^{(t)}, y, 1_R^{(n-t-1)}) \in I$.

Proof. (i)⇒(ii) Suppose that $x \in P$ and $\sqrt{I}^{(m,n)} = P \cap (\bigcap_{Q_j \in Min(I)} Q_j)$. If $x \in \sqrt{I}^{(m,n)}$, then there exists $t \in \mathbb{N}$ such that $g(x^{(t)}, 1_R^{(n-t)}) \in I$ for $t \leq n$, or $g_{(l)}(x^{(t)}) \in I$ for $t = l(n-1)+1$. If we choose $y = 1_R$, then the claim follows.

Now let $x \in P \setminus \sqrt{I}^{(m,n)}$. We may assume that $x \in P \cap (\bigcap_{j=1}^{s} Q_i)$ but $x \notin \bigcup_{j \geq s+1} Q_j$. Let $w \in \bigcap_{j \geq s+1} Q_j \setminus P$, then $g(x, w, 1_R^{(n-2)}) \in P \cap (\bigcap_{j=1}^{s} Q_j) \cap (\bigcap_{j \geq s+1} Q_j)$ $= \sqrt{I}^{(m,n)}$. Hence there exists $t \in N$ such that $g(g(x, w, 1_R^{(n-2)})^{(t)}, 1_R^{(n-t)}) \in I$ for $t \leq n$, or $g_{(l)}(g(x, w, 1_R^{(n-2)})^{(t)}) \in I$ for $t = l(n-1)+1$. If $g(g(x, w, 1_R^{(n-2)})^{(t)}, 1_R^{(n-t)}) \in I$ for $t \leq n$, then $g(x^{(t)}, g(w, 1_R^{(n-1)})^{(t)}, 1_R^{(n-2t)}) \in I$ and so

$$g(x^{(t)}, g(g(w, 1_R^{(n-1)})^{(t)}, 1_R^{(n-t)}), 1_R^{(n-t-1)}) \in I.$$

We may assume $g(g(w, 1_R^{(n-1)})^{(t)}, 1_R^{(n-t)}) = y$. Thus $g(x^{(t)}, y, 1_R^{(n-t-1)}) \in I$. If $g_{(l)}(g(x, w, 1_R^{(n-2)})^{(t)}) \in I$ for $t = l(n-1)+1$, and the claim follows by using a similar argument to the previous part and [1, Lemma 4.26].

(ii)⇒(i) Let P is not a minimal n-ary prime hyperideal of I. Then there exists a minimal n-ary prime hyperideal Q of I such that $I \subseteq Q \subsetneqq P$. We choose $x \in P \setminus Q$. Hence there exist $y \in R \setminus P$ and $t \in \mathbb{N}$ such that $g(x^{(t)}, y, 1_R^{(n-t-1)}) \in I \subseteq Q$. Since Q is an n-ary prime hyperideal, then $x \in Q$ or $y \in Q$ which is a contradiction. □

THEOREM 3.6. *Suppose that I is an n-ary 2-absorbing hyperideal of a Krasner (m,n)-hyperring (R, f, g) with the scalar identity 1_R. Then there are at most two n-ary prime hyperideals of R that are minimal over I.*

Proof. Suppose that $S = \{P_i \mid P_i$ is an n-ary prime hyperideal of R that is minimal over $I\}$ and suppose that S has at least three elements. Let $P_1, P_2 \in S$ be two distinct n-ary prime hyperideals. Then there is an $x_1 \in P_1 \setminus P_2$, and there is an $x_2 \in P_2 \setminus P_1$. First we show that $g(x_1, x_2, 1_R^{(n-2)}) \in I$. By Lemma 3.5, there are $y_2 \notin P_1$ and $y_1 \notin P_2$ such that $g(x_1^{t_1}, y_2, 1_R^{(n-t_1-1)}) \in I$ and $g(x_2^{t_2}, y_1, 1_R^{(n-t_2-1)} \in I$ for some $t_1, t_2 \geqslant 1$. Since $x_1, x_2 \notin P_1 \cap P_2$ and I is an n-ary 2-absorbing hyperideal of R, we have $g(x_1, y_2, 1_R^{(n-2)}) \in I$ and $g(x_2, y_1, 1_R^{(n-2)}) \in I$. Since $x_1, x_2 \notin P_1 \cap P_2$ and $g(x_1, y_2, 1_R^{(n-2)}), g(x_2, y_1, 1_R^{(n-2)}) \in I \subseteq P_1 \cap P_2$, we have $y_2 \in P_2 \setminus P_1$ and $y_1 \in P_1 \setminus P_2$, and hence $y_1, y_2 \notin P_1 \cap P_2$. Since $g(x_1, y_2, 1_R^{(n-2)}) \in I$ and $g(x_2, y_1, 1_R^{(n-2)}) \in I$, we have

$$g(x_1, x_2, f(y_1, y_2, 0^{(m-2)}), 1_R^{n-3}) = f(g(x_1, x_2, y_2, 1_R^{(n-3)}), g(x_1, x_2, y_1, 1_R^{(n-3)}), 0^{(m-2)}) \subseteq I$$

It is clear that $f(y_1, y_2, 0^{(m-2)}) \nsubseteq P_1$ and $f(y_1, y_2, 0^{(m-2)}) \nsubseteq P_2$.

Since $g(x_1, f(y_1, y_2, 0^{(m-2)}), 1_R^{(n-2)}) \nsubseteq P_2$ and $g(x_2, f(y_1, y_2, 0^{(m-2)}), 1_R^{(n-2)}) \nsubseteq P_1$, we have $g(x_1, f(y_1, y_2, 0^{(m-2)}), 1_R^{(n-2)}) \nsubseteq I$ and $g(x_2, f(y_1, y_2, 0^{(m-2)}), 1_R^{(n-2)}) \nsubseteq I$ and hence $g(x_1, x_2, 1_R^{(n-2)}) \in I$.

Now assume there is a $P_3 \in S$ such that $P_3 \neq P_1$ and $P_3 \neq P_2$. Then we can choose $z_1 \in P_1 \setminus (P_2 \cup P_3)$, $z_2 \in P_2 \setminus (P_1 \cup P_3)$, and $z_3 \in P_3 \setminus (P_1 \cup P_2)$. By the previous argument $g(z_1, z_2, 1_R^{(n-2)}) \in I$. Since $I \subseteq P_1 \cap P_2 \cap P_3$ and $g(z_1, z_2, 1_R^{(n-2)}) \in I$, we conclude that either $z_1 \in P_3$ or $z_2 \in P_3$ which is a contradiction. Thus S has at most two elements. $\square$

THEOREM 3.7. *Suppose that I be an n-ary 2-absorbing hyperideal of a Krasner (m,n)-hyperring (R, f, g) with the scalar identity 1_R. Then one of the following statements must hold:*
(i) $\sqrt{I}^{(m,n)} = P$ *is an n-ary prime hyperideal of R such that* $g(P^{(2)}, 1_R^{(n-2)}) \subseteq I$.
(ii) $\sqrt{I}^{(m,n)} = P_1 \cap P_2$, $g(P_1, P_2, 1_R^{(n-2)}) \subseteq I$ *and* $g((\sqrt{I}^{(m,n)})^{(2)}, 1_R^{(n-2)}) \subseteq I$ *where* P_1, P_2 *are the only distinct n-ary prime hyperideals of R that are minimal over I.*

Proof. By Theorem 3.6, we have hat either $\sqrt{I}^{(m,n)}$=P is an n-ary prime hyperideal of R or $\sqrt{I}^{(m,n)} = P_1 \cap P_2$ where P_1, P_2 are the only distinct n-ary prime hyperideals of R that are minimal over I. First assume that $\sqrt{I}^{(m,n)} = P$ is an n-ary prime hyperideal of R. Let $x, y \in P$. By Theorem 3.4, we conclude that $g(x^{(2)}, 1_R^{(n-2)}), g(y^{(2)}, 1_R^{(n-2)}) \in I$. Thus

$$g(x, f(x, 0^{(m-2)}, y), y, 1_R^{(n-3)}) = f(g(x^{(2)}, y, 1_R^{(n-3)}), g(x, y^{(2)}, 1_R^{(n-3)}), 0^{(m-2)}) \subseteq I$$

Since I is an n-ary 2-absorbing hyperideal, we have

$$g(x, f(x, 0^{(m-2)}, y), 1_R^{(n-2)}) = f(g(x^{(2)}, 1_R^{(n-2)}), g(x, y, 1_R^{(n-2)}), 0^{(m-2)}) \subseteq I$$

$$\Longrightarrow g(x, y, 1_R^{(n-2)}) \in f(-g(x^{(2)}, 1_R^{(n-2)}), 0^{(m-1)}) = -f(g(x^{(2)}, 1_R^{(n-2)}), 0^{(m-1)}) \subseteq I$$

or $$g(f(x, 0^{(m-2)}, y), y, 1_R^{(n-2)}) = f(g(x, y, 1_R^{(n-2)}), g(y^{(2)}, 1_R^{(n-2)}), o^{(m-2)}) \subseteq I$$

$$\Longrightarrow g(x, y, 1_R^{(n-2)}) \in f(-g(y^{(2)}, 1_R^{(n-2)}), 0^{(m-1)}) = -f(g(y^{(2)}, 1_R^{(n-2)}), 0^{(m-1)}) \subseteq I$$

or $$g(x, y, 1_R^{(n-2)}) \in I.$$

Hence $g(P^{(n-2)}, 1_R^{(n-2)}) \subseteq I$.

Now assume that $\sqrt{I}^{(m,n)} = P_1 \cap P_2$ where P_1, P_2 are the only distinct n-ary prime hyperideals of R that are minimal over I. Let $x, y \in \sqrt{I}^{(m,n)}$. Then $g(x, y, 1_R^{(n-2)}) \in I$ by the same argument given above, and so $g((\sqrt{I}^{(m,n)})^{(2)}, 1_R^{(n-2)}) \subseteq I$. Let $a_1 \in P_1 \setminus P_2$ and $a_2 \in P_2 \setminus P_1$. Then $g(a_1, a_2, 1_R^{(n-2)}) \in I$ by the proof of Theorem 3.6. Let $c_1 \in \sqrt{I}^{(m,n)}$ and $c_2 \in P_2 \setminus P_1$. Choose $b_1 \in P_1 \setminus P_2$. Then $g(b_1, c_2, 1_R^{(n-2)}) \in I$ by the proof of Theorem 3.6 and $f(c_1, b_1, 0^{(m-2)}) \in P_1 \setminus P_2$. Hence

$$f(g(c_1, c_2, 1_R^{(n-2)}), g(b_1, c_2, 1^{(n-2)}), 0_R^{(m-2)}) = g(c_2, f(c_1, b_1, 0_R^{(m-2)}), 1_R^{(n-2)}) \subseteq I$$

$$\Longrightarrow g(c_1, c_2, 1_R^{(n-2)}) \in f(-g(b_1, c_2, 1^{(n-2)}), 0_R^{(m-1)}) = -f(g(b_1, c_2, 1^{(n-2)}), 0_R^{(m-1)}) \subseteq I$$

By using a similar argument, we can show that if $c_1 \in \sqrt{I}^{(m,n)}$ and $c_2 \in P_1 \setminus P_2$, then $g(c_1, c_2, 1^{(n-2)}) \in I$. Therefore $g(P_1, P_2, 1_R^{(n-2)}) \subseteq I$. $\square$

THEOREM 3.8. *Suppose that I is an n-ary primary hyperideal of a Krasner (m,n)-hyperring (R,f,g) with the scalar identity 1_R such that $\sqrt{I}^{(m,n)} = P$ for some n-ary prime hyperideal P of R. Then I is an n-ary 2-absorbing hyperideal of R if and only if $g(P^{(2)}, 1_R^{(n-2)}) \subseteq I$.*

Proof. ($\Rightarrow$) Assume that I is an n-ary 2-absorbing hyperideal of a Krasner (m,n)-hyperring (R,f,g). Then $g(P^{(2)}, 1^{(n-2)}) \subseteq I$ by Theorem 3.7 (i).

($\Leftarrow$) Assume that $g(P^{(2)}, 1^{(n-2)}) \subseteq I$ and $g(x_1^n) \in I$ for some $x_1^n \in R$. If either $x_1 \in I$ or $g(x_2^n, 1_R) \in I$ for some $x_2^n \in R$, then there is nothing to prove. Hence suppose that $x_1 \notin I$ and $g(x_2^n, 1_R) \notin I$. Since I is an n-ary primary hyperideal of R and $\sqrt{I}^{(m,n)} = P$, we conclude that $x_1 \in P$ and $g(x_2^n, 1_R) \in P$. Thus $x_1 \in P$ and there exists $2 \leq i \leq n$ such that $x_i \in P$. Since $g(P^{(2)}, 1_R^{(n-2)}) \subseteq I$, we have $g(x_1, x_i, 1_R^{(n-2)}) \in I$. Thus I is an n-ary primary hyperideal of R. □

Recall that an n-ary prime hyperideal of a Krasner (m,n)-hyperring (R,f,g) with the scalar identity 1_R is called a divided prime if $P \subset \prec x \succ$ for every $x \in R \setminus P$. (Recall that $\prec x \succ = g(R, x, 1_R^{(n-2)}) = \{g(r, x, 1_R^{(n-2)} \mid r \in R\}$.)

THEOREM 3.9. *Let P be an n-ary nonzero divided prime hyperideal of a Krasner (m,n)-hyperring (R,f,g) with the scalar identity 1_R and I be an hyperideal of R such that $\sqrt{I}^{(m,n)} = P$. Then the following statements are equivalent:*
(i) I is an n-ary 2-absorbing hyperideal of R;

(ii) I is an n-ary primary hyperideal of R such that $g(P^{(2)}, 1_R^{(n-2)}) \subseteq I$.

Proof. (i)$\Rightarrow$(ii) Assume that I is an n-ary 2-absorbing hyperideal of R. Since $\sqrt{I}^{(m,n)}$ $= P$ is an n-ary nonzero prime hyperideal of R, $g(P^{(2)}, 1_R^{(n-2)}) \subseteq I$ by Theorem 3.7 (i). Now let $g(x_1^n) \in I$ for some $x_1^n \in R$ and assume that $g(x_1^{i-1}, 1_R, x_{i+1}^n) \notin P$. Since $x_i {\in} P$ and P is a divided hyperideal of R, we have $x_i{=}g(r, g(x_1^{i-1}, 1_R, x_{i+1}^n), 1_R^{(n-2)})$ for some $r \in R$. Thus $g(x_1^n) = g(x_1^{i-1}, g(r, g(x_1^{i-1}, 1_R, x_{i+1}^n), 1_R^{(n-2)}), x_{i+1}^n) \in I$. Since I is an n-ary 2-absorbing hyperideal of R, we have
$$g(x_j, g(r, g(x_1^{i-1}, 1_R, x_{i+1}^n), 1_R^{(n-2)}), 1_R^{(n-2)})$$
$$= g(x_j, r, g(g(x_1^{i-1}, 1_R, x_{i+1}^n), 1_R^{(n-1)}), 1_R^{(n-3)}) = g(x_j, r, g(x_1^{i-1}, 1_R, x_{i+1}^n), 1_R^{(n-3)}) \in I$$
for some $1 \leq j \leq i-1$ or $i+1 \leq j \leq n$. Since $g(x_j, g(x_1^{i-1}, 1_R, x_{i+1}^n), 1_R^{(n-2)}) \notin I$, we conclude that $g(r, g(x_1^{i-1}, 1_R, x_{i+1}^n), 1_R^{(n-2)}) = x_i \in I$ or $g(x_j, r, 1_R^{(n-2)}) \in I$. The second possibility implies that
$$g(x_j, r, g(x_1^{j-1}, x_{j+1}^{i-1}, x_{i+1}^n), 1_R^{(2)}), 1_R^{(n-3)}) = g(r, g(x_1^{i-1}, 1_R, x_{i+1}^n), 1_R^{(n-2)}) = x_i \in I$$
or $$g(x_j, r, g(x_1^{i-1}, x_{i+1}^{j-1}, x_{j+1}^n), 1_R^{(2)}), 1_R^{(n-3)}) = g(r, g(x_1^{i-1}, 1_R, x_{i+1}^n), 1_R^{(n-2)}) = x_i \in I.$$
Hence I is an n-ary primary hyperideal of R.

(ii)$\Rightarrow$(i) This follows directly from Theorem 3.8. □

4. Extensions of n-ary 2-absorbing hyperideals

In this section, we investigate the stabilty of n-ary 2-absorbing hyperideals in some hyperring-theoretic constructions. We start with an example.

Example 4.1. Consider the Krasner (m,n)-hyperring $(\bar{R},f,g)$ constructed in [1, Example 2.4]. The hyperideal of $\bar{R}$ is of the form $\bar{I}$ such that $G \subset \bar{I} \lhd \bar{R}$. If I is a prime ideal of R such that $G \subset I$, then the n-ary 2-absorbing hyperideals of $\bar{R}$ are of the form $\bar{I}$.

Let for $\bar{x}_1^n \in \bar{R}$, $g(\bar{x}_1,\ldots,\bar{x}_1) \in \bar{I}$. Then we have $x_1,\ldots,x_n \in I$. Since I is a 2-absorbing ideal, we conclude that $x_ix_j \in I$ for some $1 \leq \prec j \leq n$. Therefore we have $x_ix_j(1_R)^{(n-2)} \in I$ and so $g(x_i,x_j,1_{\bar{R}}^{n-2)}) \in \bar{I}$. Hence $\bar{I}$ is an n-ary 2-absorbing hyperideal of $(\bar{R},f,g)$.

Theorem 4.2. *Let (R_1,f_1,g_1) and (R_2,f_2,g_2) be two Krasner (m,n)-hyperrings and $\phi : R_1 \longrightarrow R_2$ be a homomorphism. Then the following statements hold:*
(i) If I_2 is an n-ary 2-absorbing hyperideal of R_2, then $\phi^{-1}(I_2)$ is an n-ary 2-absorbing hyperideal of R_1.

(ii) If ϕ is an epimorphism and I_1 is an n-ary 2-absorbing hyperideal of R_1 containing $Ker(\phi)$, then $\phi(I_1)$ is an n-ary 2-absorbing hyperideal of R_2.

Proof. (i) Let $g_1(x_1^n) \in \phi^{-1}(I_2)$ for some $x_1^n \in R_1$. Then $\phi(g_1(x_1^n)) = g_2(\phi(x_1^n)) \in I_2$. Since I_2 is a 2-absorbing hyperideal of R_2, there exist i,j, $1 \leq i \prec j \leq n$ such that $g_2(\phi(x_i),\phi(x_j),1_{R_2}^{(n-2)}) \in I_2$. Thus $\phi^{-1}(g_2(\phi(x_i),\phi(x_j),1_{R_2}^{(n-2)})) = g_1(x_i,x_j,1_{R_1}^{(n-2)}) \in \phi^{-1}(I_2)$. Hence $\phi^{-1}(I_2)$ is an n-ary 2-absorbing hyperideal of R_1.

(ii) Let $g_2(y_1^n) \in \phi(I_1)$ for some $y_1^n \in R_2$. Since ϕ is an epimorphism, then there exists $x_1^n \in R_1$ such that $\phi(x_t) = y_t$ for all t, $1 \leq t \leq n$ and $\phi(g_1(x_1^n)) = g_2(\phi(x_1^n)) = g_2(y_1^n) \subseteq \phi(I_1)$. Since $Ker\ \phi \subseteq I_1$, we have $g_1(x_1^n) \in I_1$. Since I_1 is a 2-absorbing hyperideal of R_1, then there exist i,j, $1 \leq i \prec j \leq n$ such that $g_1(x_i,x_j,1_{R_1}^{(n-2)}) \in I_1$. This implies that $\phi(g_1(x_i,x_j,1_{R_1}^{(n-2)})) = g_2(\phi(x_i),\phi(x_j),1_{R_2}^{n-2)}) = g_2(y_i,y_j,1_{R_2}^{(n-2)}) \in \phi(I_1)$. Thus $\phi(I_1)$ is an n-ary 2-absorbing hyperideal of R_2. $\square$

Theorem 4.3. *Let I be an n-ary 2-absorbing hyperideal of a Krasner (m,n)-hyperring (R,f,g) with the scalar identity 1_R. If J is a hyperideal of R such that $J \subseteq I$, then I/J is an n-ary 2-absorbing hyperideal of R/J.*

Proof. Suppose $g(f(x_{11}^{1(i-1)},J,x_{1(i+1)}^{1m}),\ldots,f(x_{n1}^{n(i-1)},J,x_{n(i+1)}^{nm})) \in I/J$ for some $x_{11}^{1m},\ldots,x_{m1}^{mm} \in R/J$. Thus $f(g(x_{11}^{n1}),\ldots,g(x_{1(i-1)}^{n(i-1)}),J,g(x_{1(i+1)}^{n(i+1)}),\ldots,g(x_{1m}^{nm})) \in I/J$. Hence

$$f(g(x_{11}^{n1}),\ldots,g(x_{1(i-1)}^{n(i-1)}),0,g(x_{1(i+1)}^{n(i+1)}),\ldots,g(x_{1m}^{nm})) \subseteq I$$

$$\Longrightarrow g(f(x_{11}^{1(i-1)},0,x_{1(i+1)}^{1m}),\ldots,f(x_{n1}^{n(i-1)},0,x_{n(i+1)}^{nm}) \subseteq I.$$

Since I is an n-ary 2-absorbing hyperideal, then there exist $1 \leq s \prec t \leq n$ such that $g(f(x_{s1}^{s(i-1)}, 0, x_{s(i+1)}^{sm}), f(x_{t1}^{t(i-1)}, 0, x_{t(i+1)}^{tm}), 1_R^{(n-2)}) \subseteq I$. Therefore

$$\begin{aligned} &f(g(f(x_{s1}^{s(i-1)}, 0, x_{s(i+1)}^{sm}), f(x_{t1}^{t(i-1)}, 0, x_{t(i+1)}^{tm}), 1_R^{(n-2)}), J, 0^{(m-2)}) \in I/J \\ &\Longrightarrow g(f(x_{s1}^{s(i-1)}, J, x_{s(i+1)}^{sm}), f(x_{t1}^{t(i-1)}, J, x_{t(i+1)}^{tm}), 1_{R/J}^{(n-2)}) \in I/J. \end{aligned}$$

Thus I/J is an n-ary 2-absorbing hyperideal of R/J. □

Let (R_1, f_1, g_1) and (R_2, f_2, g_2) be two Krasner (m,n)-hyperrings such that 1_{R_1} and 1_{R_2} be scalar identities of R_1, R_2, respectively. Then the (m,n)-hyperring $(R_1 \times R_2, f_1 \times f_2, g_1 \times g_2)$ is defined by m-ary hyperoperation $f_1 \times f_2$ and n-ary hyperoperation $g_1 \times g_2$, as follows:

$$\begin{aligned} f_1 \times f_2((a_1,b_1), \ldots, (a_m,b_m)) &= \{(a,b) \mid a \in f_1(a_1^m), b \in f_2(b_1^m)\} \\ g_1 \times g_2((x_1,y_1), \ldots, (x_n,y_n)) &= (g_1(x_1^n), g_2(y_1^n)), \end{aligned}$$

for all $a_1^m, x_1^n \in R_1$ and $b_1^m, y_1^n \in R_2$.

Theorem 4.4. *Let (R_1, f_1, g_1) and (R_2, f_2, g_2) be two Krasner (m,n)-hyperrings such that 1_{R_1} and 1_{R_2} be scalar identities of R_1, R_2, respectively. Then the following statements hold:*
(i) I_1 is an n-ary 2-absorbing hyperideal of R_1 if and only if $I_1 \times R_2$ is an n-ary 2-absorbing hyperideal of $R_1 \times R_2$.

(ii) I_2 is an n-ary 2-absorbing hyperideal of R_2 if and only if $R_1 \times I_2$ is an n-ary 2-absorbing hyperideal of $R_1 \times R_2$.

Proof. (i) ($\Rightarrow$) Assume that I_1 is an n-ary 2-absorbing hyperideal of R_1. Let $g_1 \times g_2((x_1,y_1), \ldots, (x_n,y_n)) \in I_1 \times R_2$, with $x_1^n \in R_1$ and $y_1^n \in R_2$. Then we have $g_1(x_1^n) \in I_1$. Since I_1 is an n-ary 2-absorbing hyperideal of R_1, we conclude that there exist i, j, $1 \leq i \prec j \leq n$ such that $g(x_i, x_j, 1_{R_1}^{(n-2)}) \in I_1$. This implies that $g_1 \times g_2((x_i,y_i), (x_j,y_j), (1_{R_1}, 1_{R_2})^{(n-2)}) \in I_1 \times R_2$. Thus $I_1 \times R_2$ is an n-ary 2-absorbing hyperideal of $R_1 \times R_2$.

($\Leftarrow$) Suppose that $I_1 \times R_2$ is an n-ary 2-absorbing hyperideal of $R_1 \times R_2$. Let $g(x_1^n) \in I_1$ with $x_1^n \in R_1$. Then $g_1 \times g_2((x_1, 1_{R_2}), \ldots (x_n, 1_{R_2})) \in I_1 \times R_2$. Since $I_1 \times R_2$ is an n-ary 2-absorbing hyperideal of $R_1 \times R_2$, we conclude that there exist i, j, $1 \leq i \prec j \leq n$ such that $g_1 \times g_2((x_i, 1_{R_2}), (x_j, 1_{R_2}), (1_{R_1}, 1_{R_2})^{(n-2)}) \in I_1 \times R_2$, which means that $g_1(x_i, x_j, 1_{R_1}^{(n-2)}) \in I_1$. Thus I_1 is an n-ary 2-absorbing hyperideal of R_1.

(ii) The proof is similar to (i). □

Let I be a normal hyperideal of Krasner (m,n)-hyperring (R, f, g). Then the set of all equivalence classes $[R : I^*] = \{I^*[x] \mid x \in R\}$ is a Krasner (m,n)-hyperring with the m-ary hyperoperation f/I and the n-ary operation g/I, defined as follows:

$$\begin{aligned} f/I(I^*[x_1], \ldots, I^*[x_m]) &= \{I^*[z] \mid z \in f(I^*[x_1], \ldots, I^*[x_m])\}, \quad \forall x_1^m \in R \\ g/I(I^*[x_1], \ldots, I^*[x_n]) &= I^*[g(x_1^n)], \quad \forall x_1^n \in R \end{aligned}$$

(for more details refer to [15]).

Theorem 4.5. *Let I be a normal hyperideal and J be an n-ary 2-absorbing hyperideal, respectively, of a Krasner (m,n)-hyperring (R,f,g) with the scalar identity 1_R such that $I \subseteq J$. Then $[J : I^*]$ is an n-ary 2-absorbing hyperideal of $[R : I^*]$.*

Proof. Suppose that $g/f(I^*[x_1],\ldots,I^*[x_n]) \in [J : I^*]$ for some $x_1^n \in R$. Thus $g/f(I^*[x_1],\ldots,I^*[x_n]) = I^*[g(x_1^n)] \in [J : I^*]$. This means $I^*[g(x_1^n)] \subseteq J$. Therefore

$$\begin{aligned} I^*[g(x_1^n)] &= f(I, g(x_1^n), 0^{(m-2)}) = f(I, g(x_1^n), g(0^{(n)})^{(m-2)}) \\ &= g(f(I, x_1, 0^{(m-2)}), \ldots, f(I, x_n, 0^{(m-2)})) \subseteq J \end{aligned}$$

Since J is an n-ary 2-absorbing hyperideal of R, then we conclude that

$$g(f(I, x_i, 0^{(m-2)}), f(I, x_j, 0^{(m-2)}), 1_R^{(n-2)}) \subseteq J \text{ for some } 1 \leq i \prec j \leq n.$$

Hence

$$\begin{aligned} &g(f(I, x_i, 0^{(m-2)}), f(I, x_j, 0^{(m-2)}), f(I, 1_R, 0^{(m-2)})^{(n-2)}) \\ &= f(I, g(x_i, x_j, 1_R^{(n-2)}), g(0^{(n)})^{(m-2)}) = f(I, g(x_i, x_j, 1_R^{(n-2)}), 0^{(m-2)}) \\ &= I^*[g(x_i, x_j, 1_R^{(n-2)})] = g/I(I^*[x_i], I^*[x_j], I^*[1_R]^{(n-2)}) \in [J : I]. \end{aligned}$$

Thus $[J : I^*]$ is an n-ary 2-absorbing hyperideal of $[R : I^*]$. □

5. n-ary 2-absorbing primary hyperideals in a Krasner $(\mathbf{m}, \mathbf{n})$-hyperring

Definition 5.1. A nonzero proper hyperideal I of a Krasner (m,n)-hyperring (R,f,g) with the scalar identity 1_R is said to be n-ary 2-absorbing primary if for $x_1^n \in R$, $g(x_1^n) \in I$ implies that $g(x_1, x_2, 1_R^{(n-2)}) \in I$ or $g(x_t, x_i, 1_R^{(n-2)}) \in \sqrt{I}^{(m,n)}$ or $g(x_i, x_j, 1_R^{(n-2)}) \in \sqrt{I}^{(m,n)}$ for $t \in \{1,2\}$ and some i, j, $3 \leq i \prec j \leq n$.

Example 5.2. Let R be a Krasner (m,n)-hyperring (R,f,g) with the scalar identity 1_R. Then every n-ary primary hyperideal of R is an n-ary 2-absorbing primary hyperideal.

Let Q be an n-ary primary hyperideal of R and $g(x_1^n) \in Q$. Then either $x_i \in Q$ or $g(x_1^{i-1}, 1_R, x_{i+1}^n) \in \sqrt{Q}^{(m,n)}$. We may assume that $x_1 \in Q$ or $g(1_R, x_2^n) \in \sqrt{Q}^{(m,n)}$. Since $\sqrt{Q}^{(m,n)} = P$ is an n-ary prime hyperideal of R by Theorem 4.28 in [1], then we get $x_1 \in Q$ or $x_2 \in P$ or $\ldots$ or $x_n \in P$. Since Q and P are hyperideals of R, we get $g(x_1, x_2, 1_R^{(n-2)}) \in Q$ or $g(x_t, x_i, 1_R^{(n-2)}) \in P = \sqrt{Q}^{(m,n)}$ or $g(x_i, x_j, 1_R^{(n-2)}) \in P = \sqrt{Q}^{(m,n)}$ for $t \in \{1,2\}$ and some $3 \leq i \prec j \leq n$.

Theorem 5.3. *If I is an n-ary 2-absorbing ideal of a Krasner (m,n)-hyperring (R,f,g) with the scalar identity 1_R, then $\sqrt{I}^{(m,n)}$ is an n-ary 2-absorbing hyperideal of R.*

Proof. Let $g(x_1^n) \in \sqrt{I}^{(m,n)}$ for some $x_1^n \in R$ such that neither $g(x_t, x_i, 1_R^{(n-2)}) \in \sqrt{I}^{(m,n)}$ nor $g(x_i, x_j, 1_R^{(n-2)}) \in \sqrt{I}^{(m,n)}$ for $t \in \{1,2\}$ and all i, j, $3 \leq i \prec j \leq n$. Then there exists $t \in \mathbb{N}$ such that $g(g(x_1^n)^{(t)}, 1_R^{(n-t)}) \in I$ for $t \leq n$, or $g_{(l)}(g(x_1^n)^{(t)}) \in I$ for $t \succ n$, $t = l(n-1)+1$. If $g(g(x_1^n)^{(t)}, 1_R^{(n-t)}) \in I$ for $t \leq n$, then we have $g(g(x_1^{(t)}, 1_R^{(n-t}), \dots, g(x_n^{(t)}, 1_R^{(n-t)}) {\in} I$. Since I is an n-ary 2-absorbing primary of R, we obtain that $g(g(x_1^{(t)}, 1_R^{(n-t)}), g(x_2^{(t)}, 1_R^{(n-t)}), 1_R^{(n-2)}) {=} g(g(x_1, x_2, 1_R^{(n-2)})^{(t)}, 1_R^{(n-t)}) \in I$.

This means that $g(x_1, x_2, 1_R^{(n-2)}) \in \sqrt{I}^{(m,n)}$. Similarly for the other case. Therefore $\sqrt{I}^{(m,n)}$ is an n-ary 2-absorbing hyperideal of R. □

THEOREM 5.4. *If I is an n-ary 2-absorbing primary of a Krasner (m,n)-hyperring (R,f,g) with the scalar identity 1_R, then either $\sqrt{I}^{(m,n)} = P$ such that P is an n-ary prime hyperideal of R or $\sqrt{I}^{(m,n)} = P_1 \cap P_2$ such that P_1, P_2 are the only distinct n-ary prime hyperideals of R that are minimal over I.*

Proof. This follows from Theorems 5.3 and 3.7. □

THEOREM 5.5. *Let I_1 and I_2 be n-ary P_1-primary and P_2-primary hyperideals of (R,f,g) respectively such that P_1 and P_2 are two n-ary prime hyperideals of (R,f,g). Then $I = I_1 \cap I_2$ is an n-ary 2-absorbing primary hyperideals.*

Proof. It is clear that $\sqrt{I}^{(m,n)} {=} \sqrt{I_1}^{(m,n)} \cap \sqrt{I_2}^{(m,n)} {=} P_1 \cap P_2$. Assume that $g(x_1^n) \in I$ for some $x_1^n \in R$ such that neither $g(x_t, x_i, 1_R^{(n-2)}) \in \sqrt{I}^{(m,n)}$ nor $g(x_i, x_j, 1_R^{(n-2)}) \in \sqrt{I}^{(m,n)}$ for $t \in \{1,2\}$ and all i,j, $3 \leq i \prec j \leq n$. Then we have $x_1^n \notin \sqrt{I}^{(m,n)} = P_1 \cap P_2$. Since $\sqrt{I}^{(m,n)}$ is an n-ary 2-absorbing hyperideal of R, we obtain $g(x_1, x_2, 1_R^{(n-2)}) \in \sqrt{I}^{(m,n)} = P_1 \cap P_2$. It implies that $g(x_1, x_2, 1_R^{(n-2)}) \in P_1$. Since P_1 is an n-ary prime hyperideal of R, then $x_1 \in P_1$ or $x_2 \in P_1$. We may suppose that $x_1 \in P_1$. Also, $g(x_1, x_2, 1_R^{(n-2)}) \in P_2$ which implies that $x_2 \in P_2$ but $x_1 \notin P_2$. Since $x_2 \in P_2$, $x_2 \notin \sqrt{I}^{(m,n)}$, then we have $x_2 \notin P_1$. If $x_1 \in I_1$ and $x_2 \in I_2$, then we are done. Hence we suppose that $x_1 \notin I_1$. Then we have $g(1_R, x_2, x_3, \dots, x_n) \in P_1$, because I_1 is an n-ary primary hyperideal of R. Since $x_2 \in P_2$ and $g(1_R, x_2, x_3, \dots, x_n) \in P_1$, we get $g(1_R, x_2, x_3, \dots, x_n) \in P_1 \cap P_2 = \sqrt{I}^{(m,n)}$. This ia a contradiction. Thus $x_1 \in I_1$. By using a similar argument, we have $x_2 \in I_2$. Thus $g(x_1, x_2, 1^{(n-2)}) \in I = I_1 \cap I_2$. □

THEOREM 5.6. *Let I be a hyperideal of R such that $\sqrt{I}^{(m,n)}$ is an n-ary prime hyperideal of (R,f,g). Then I is an n-ary 2-absorbing primary hyperideal of R.*

Proof. Let $g(x_1^n) \in I$ for some $x_1^n \in R$ such that $g(x_1, x_2, 1^{(n-2)}) \notin I$. Thus we have $g(g(x_1^n), g(1_R^{(2)}, x_3^n)^{(2)}, 1_R^{(n-3)}) {\in} I {\subseteq} \sqrt{I}^{(m,n)}$, which implies $g(g(x_1, 1_R, x_3^n), g(1_R, x_2^n), x_3^n) {\in} \sqrt{I}^{(m,n)}$. Since $\sqrt{I}^{(m,n)}$ is an n-ary prime hyperideal of R, then we obtain $g(x_1, 1_R, x_3^n) {\in} \sqrt{I}^{(m,n)}$ or $g(1_R, x_2^n) \in \sqrt{I}^{(m,n)}$ or $x_3 \in \sqrt{I}^{(m,n)}$ or …or $x_n \in \sqrt{I}^{(m,n)}$. It means that $g(x_t, x_i, 1_R^{(n-2)}) \in \sqrt{I}^{(m,n)}$ or $g(x_i, x_j, 1_R^{(n-2)}) \in \sqrt{I}^{(m,n)}$ for $t \in \{1,2\}$ and some $3 \leq i \prec j \leq n$, because $\sqrt{I}^{(m,n)}$ is a hyperideal of R. □

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