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INTERPOLATIVE HARDY-ROGERS AND REICH-RUS-ĆIRIĆ TYPE CONTRACTIONS IN b-METRIC SPACES AND RECTANGULAR b-METRIC SPACES

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Abstract. In this paper, we investigate two fixed point theorems in the framework of b-metric spaces and rectangular b-metric spaces, using interpolative approach. One is Hardy-Rogers and the other is Reich-Rus-Ćirić type contraction. Examples are provided in support of the results.

1. Introduction

After Banach proved his celebrated contraction principle in 1922, numerous researchers have tried to improve or generalise it. The generalisations were mainly done in two directions—either the contractive condition was replaced by some more general ones, or the scope of the metric space was broadened. But not all of such attempts in generalizations were useful in applications, which was the main motive of such investigations. Some of them were not even true generalizations as they appeared to be equivalent to already existing ones.

Among the many generalised versions of metric spaces, the one due to Bakhtin [4] and Czerwik [6], called b-metric space, has drawn attention of researchers all over the world due to its importance and ease of applicability in many fields. The initial motivation for the introduction of b-metric space was to study the issue of convergence of measurable functions in regard to measure. Since then immense development of fixed point theory in the framework of b-metric space has taken place [1–3, 5, 7, 13].

In the current paper, our aim is to study two significant interpolative contractions in the framework of b-metric and rectangular b-metric spaces where real and non-trivial generalisations are possible and as such, application of the results in relevant fields becomes feasible and easier.

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2. Preliminaries

First, we list some important definitions and relevant theorems, which are useful in our main results. Throughout the paper $\mathbb{N}$, $\mathbb{R}$ and $\mathbb{R}_+$ denote the set of natural numbers, set of real numbers and set of nonnegative real numbers respectively.

Definition 2.1 ([4,6]). Let U be a nonempty set and the mapping $h : U \times U \to [0, \infty)$ satisfies:
(bM1) $h(u, v) = 0$ if and only if $u = v$;

(bM2) $h(u, v) = h(v, u)$ for all $u, v \in U$;

(bM3) there exists a real number $s \geq 1$ such that $h(u, z) \leq s[h(u, v) + h(v, z)]$ for all $u, v, z \in U$.
Then h is called a b-metric on U and (U, h) is called a b-metric space (in short bMS) with coefficient s.

Definition 2.2 ([8]). Let U be a nonempty set and the mapping $h : U \times U \to [0, \infty)$ satisfies:
(RbM1) $h(u, v) = 0$ if and only if $u = v$;

(RbM2) $h(u, v) = h(v, u)$ for all $u, v \in U$;

(RbM3) there exists a real number $s \geq 1$ such that $h(u, v) \leq s[h(u, w) + h(w, z) + h(z, v)]$ for all $u, v \in U$ and all distinct points $w, z \in U \backslash \{u, v\}$.
Then h is called a rectangular b-metric on U and (U, h) is called a rectangular b-metric space (in short RbMS) with coefficient s.

Definition 2.3 ([8]). Let (U, h) be a (rectangular) b-metric space, $\{u_n\}$ be a sequence in U and $u \in U$. Then
(a) The sequence $\{u_n\}$ is said to be convergent in (U, h) and converges to u, if for every $\varepsilon > 0$ there exists $n_0 \in \mathbb{N}$ such that $h(u_n, u) < \varepsilon$ for all $n > n_0$ and this fact is represented by $\lim\limits_{n\to\infty} u_n = u$ or $u_n \to u$ as $n \to \infty$.

(b) The sequence $\{u_n\}$ is said to be Cauchy sequence in (U, h) if for every $\varepsilon > 0$ there exists $n_0 \in \mathbb{N}$ such that $h(u_n, u_{n+p}) < \varepsilon$ for all $n > n_0, p > 0$.

(c) (U, h) is said to be a complete (rectangular) b-metric space if every Cauchy sequence in U converges to some $u \in U$.

Lemma 2.4 ([12, Lemma 1.12]). *Let (U, h) be a $b_v(s)$-metric space and $\{u_n\}$ a sequence in U such that u_n's $(n \geq 0)$ are all different. Suppose that $\lambda \in [0, 1)$ and c_1, c_2 are real nonnegative numbers such that*

$$h(u_m, u_n) \leq \lambda h(u_{m-1}, u_{n-1}) + c_1\lambda^m + c_2\lambda^n, \text{ for all } m, n \in \mathbb{N}.$$

Then $\{u_n\}$ is a Cauchy sequence.

Recently, some interesting fixed point results for interpolative contractions have been studied in [9, 10].

3. Results in b-metric spaces

First we define Hardy-Rogers type contraction in a b-metric space and discuss the corresponding fixed point theorem.

DEFINITION 3.1. Let (U,h,s) be a (rectangular) b-metric space. The self-map $T: U \to U$ is called an interpolative Hardy-Rogers type contraction map if there exist $\lambda \in [0,1)$ and $p,q,r \in (0,1)$ with $p+q+r<1$ such that

$$h(Tu,Tv) \le \lambda[h(u,v)]^q[h(u,Tv)]^p[h(v,Tv)]^r[\frac{1}{2s}(h(u,Tv)+h(v,Tu))]^{1-p-q-r} \quad (1)$$

for all $u,v \in U \setminus Fix(T)$.

THEOREM 3.2. *Let (U,h,s) be a complete b-metric space and T be an interpolative Hardy-Rogers type contraction. Then T has a fixed point.*

Proof. Let $u_0 \in U$ and define the sequence $\{u_n\}$ by $u_n = T^n(u_0)$ for all $n \in \mathbb{N}$. If there exists $n_0 \in \mathbb{N}$ such that $u_{n_0} = u_{n_0+1}$, then u_{n_0} becomes a fixed point of T. So assume that $u_n \neq u_{n+1}$ for all $n \ge 0$.

Replacing u by u_n and v by u_{n-1} in (1), we obtain

$$\begin{aligned} h(u_{n+1},u_n) &= d(Tu_n,Tu_{n-1}) \\ &\le \lambda[h(u_n,u_{n-1})]^q[h(u_n,Tu_n)]^p[h(u_{n-1},Tu_{n-1})]^r[\frac{1}{2s}(h(u_n,u_n)+h(u_{n-1},u_{n+1}))]^{1-p-q-r} \\ &\le \lambda[h(u_n,u_{n-1})]^q[h(u_n,u_{n+1})]^p[h(u_{n-1},u_n)]^r[\frac{1}{2s}.s(h(u_{n-1},u_n)+h(u_n,u_{n+1}))]^{1-p-q-r}. \end{aligned} \quad (2)$$

If possible, let $h(u_{n-1},u_n) < h(u_n,u_{n+1})$ for some $n \ge 1$. Then $\frac{1}{2}[h(u_{n-1},u_n) + h(u_n,u_{n+1})] \le h(u_n,u_{n+1})$. Thus from (2) we have

$$\begin{aligned} h(u_{n+1},u_n) &\le \lambda[h(u_n,u_{n-1})]^q[h(u_n,u_{n+1})]^p[h(u_{n-1},u_n)]^r[(h(u_n,u_{n+1})]^{1-p-q-r} \\ &= \lambda[h(u_n,u_{n-1})]^{q+r}[h(u_n,u_{n+1})]^{1-q-r}. \end{aligned}$$

This implies $[h(u_n,u_{n+1})]^{q+r} \le \lambda[h(u_{n-1},u_n)]^{q+r}$. So we must have $h(u_n,u_{n+1}) \le h(u_{n-1},u_n)$, which is a contradiction. Hence

$$h(u_n,u_{n+1}) < h(u_{n-1},u_n) \text{ for all } n \ge 1. \quad (3)$$

Now from (2) we have

$$\begin{aligned} h(u_{n+1},u_n) &\le \lambda[h(u_n,u_{n-1})]^q[h(u_n,u_{n+1})]^p[h(u_{n-1},u_n)]^r[d(u_{n-1},u_n))]^{1-p-q-r} \\ &= \lambda[h(u_{n-1},u_n)]^{1-p}[h(u_n,u_{n+1})]^p. \end{aligned}$$

This implies

$$[h(u_n,u_{n+1})]^{1-p} \le \lambda[h(u_{n-1},u_n)]^{1-p} \text{ for all } n \ge 1. \quad (4)$$

Combining (3) and (4) we conclude that

$$h(u_n,u_{n+1}) \le \lambda h(u_{n-1},u_n) \text{ for all } n \ge 1. \quad (5)$$

It has been proved in [11] that every sequence $\{u_n\}$ in a b-metric space (U,h,s) having the property that there exists $\lambda \in [0,1)$ such that $h(u_n,u_{n+1}) \le \lambda h(u_{n-1},u_n)$

for all $n \geq 1$, is a Cauchy sequence. Thus from (5), we conclude that $\{u_n\}$ is a Cauchy sequence. Again (U, h, s) being complete, there exists $l \in U$ such that $\lim_{n\to\infty} u_n = l$. Since $u_n \neq Tu_n$ for all $n \geq 0$, replacing u by u_n and v by l in (1), we have

$$\begin{aligned} h(u_{n+1}, Tl) &= h(Tu_n, Tl) \\ &\leq \lambda [h(u_n, l)]^q [h(u_n, Tu_n)]^p [h(l, Tl)]^r [\frac{1}{2s}(h(u_n, Tl) + h(l, Tu_n))]^{1-p-q-r} \end{aligned} \tag{6}$$

Taking limit as $n \to \infty$ in (6), we have $h(l, Tl) = 0$, which implies that $Tl = l$. □

EXAMPLE 3.3. Let $U = [0, \infty)$ and $h(u, v) = (u-v)^2$. Then it is a routine verification that $(U, h, 2)$ is a complete b-metric space. Define the self-map T on U by

$$Tu = \begin{cases} 0, & \text{if } u \in [0, 1), \\ u, & \text{if } u \geq 1. \end{cases}$$

Let $u, v \in U \setminus Fix(T)$. Then clearly $u, v \in (0, 1)$. Now $h(Tu, Tv) = h(0, 0) = 0$, i.e., (2) holds. Therefore, all hypotheses of Theorem 3.2 hold and thus T has a fixed point. Here, it is easy to see that T has infinitely many fixed points.

Next we prove a fixed point theorem for Reich-Rus-Ćirić type contraction in b-metric spaces.

DEFINITION 3.4. Let (U, h, s) be a (rectangular) b-metric space. A self-map $T : U \to U$ is called an interpolative Reich-Rus-Ćirić type contraction if there are constants $\lambda \in [0, 1)$ and $A, B \in (0, 1)$ with $A + B < 1$ such that

$$h(Tu, Tv) \leq \lambda [h(u, v)]^B [h(u, Tu)]^A [(h(v, Tv)]^{1-A-B} \tag{7}$$

for all $u, v \in U \setminus Fix(T)$.

THEOREM 3.5. *Let (U, h, s) be a complete b-metric space. If $T : U \to U$ is an interpolative Reich-Rus-Ćirić type contraction, then T has a fixed point in U.*

Proof. Let $u_0 \in U$ and define the iterative sequence $\{u_n\}$ by $u_n = T^n u_0$ for all $n \in \mathbb{N}$. If there exists $n_0 \in \mathbb{N}$ such that $u_{n_0} = u_{n_0+1}$, then u_{n_0} is clearly a fixed point of T and the proof is complete. So assume that $u_n \neq u_{n+1}$ for all $n \geq 0$.

Substituting u by u_n and v by u_{n-1} in (7), we have

$$\begin{aligned} h(u_{n+1}, u_n) &= h(Tu_n, Tu_{n-1}) \leq \lambda [h(u_n, u_{n-1})]^B [h(u_n, Tu_n)]^A [h(u_{n-1}, Tu_{n-1})]^{1-A-B} \\ &= \lambda [h(u_n, u_{n-1})]^B [h(u_n, u_{n+1})]^A [h(u_{n-1}, u_n)]^{1-A-B} \\ &\leq \lambda [h(u_n, u_{n-1})]^B [h(u_n, u_{n+1})]^A [h(u_{n-1}, u_n)]^{1-A-B} (\text{ for } s \geq 1) \\ &= \lambda [h(u_n, u_{n-1})]^{1-A} [h(u_n, u_{n+1})]^A \end{aligned} \tag{8}$$

From the above, we obtain $[h(u_n, u_{n+1})]^{1-A} \leq \lambda [h(u_n, u_{n-1})]^{1-A}$, which implies that $h(u_n, u_{n+1}) \leq h(u_n, u_{n-1})$ for all $n \geq 0$. Using these two inequalities, we have

$$h(u_n, u_{n+1}) \leq \lambda^{\frac{1}{1-A}} h(u_{n-1}, u_n) \text{ for all } n \geq 1. \tag{9}$$

But we know from [11] that every sequence $\{u_n\}$ in a b-metric space (U, d, s) satisfying the property (9) is a Cauchy sequence. Thus we conclude that $\{u_n\}$ is a Cauchy sequence and (U, h, s) being complete, there exists $\xi \in U$ such that $\lim_{n\to\infty} u_n = \xi$.

Next we show that ξ is a fixed point of T. If possible assume that $T\xi \neq \xi$ so that $d(T\xi, \xi) > 0$. Also by hypothesis, $u_n \neq Tu_n$ for all $n \geq 0$. By substituting u by u_n and v by ξ in (7), we have

$$\begin{aligned} h(\xi, T\xi) &\leq s[h(\xi, u_{n+1}) + h(Tu_n, T\xi)] \\ &\leq sh(\xi, u_{n+1}) + \lambda s[h(u_n, \xi)]^B [h(u_n, Tu_n)]^A [h(\xi, T\xi)]^{1-A-B} \\ &= sh(\xi, u_{n+1}) + \lambda s[h(u_n, \xi)]^B [h(u_n, u_{n+1})]^A [h(\xi, T\xi)]^{1-A-B}. \end{aligned} \tag{10}$$

Taking limit as $n \to \infty$ in (10), we have $h(\xi, T\xi) = 0$, which contradicts our last hypothesis. Hence $T\xi = \xi$. □

EXAMPLE 3.6. Let $U = \{0, 1, 2\}$ and $h : U \times U \to [0, \infty)$ be defined as $h(u, v) = 0$, $h(u, v) = h(v, u)$ for all $u, v \in U$, $h(0, 1) = 1$, $h(0, 2) = 2.2$ and $h(1, 2) = 1.1$. Then it is easy to see that $(U, h, \frac{22}{21})$ is a complete b-metric space (but it is not a metric space).

Define the self-map T on U by

$$Tu = \begin{cases} 0, & \text{if } u \neq 2, \\ 1, & \text{if } u = 2. \end{cases}$$

Further we can see that

$$h(Tu, Tv) = \begin{cases} h(0,0) = 0, & \text{if } u \neq 2, v \neq 2, \\ h(1,0) = 1, & \text{if } u = 2, v \neq 2, \\ h(0,1) = 1, & \text{if } u \neq 2, v = 2, \\ h(1,1) = 0, & \text{if } u = 2, v = 2. \end{cases}$$

Let $u, v \in U \setminus Fix(T)$. Then clearly the maximum value of $h(Tu, Tv)$ is 1, i.e., inequality (7) and all hypotheses of Theorem 3.5 hold if we choose $\lambda = 0.01, A = \frac{1}{2}, B = \frac{1}{3}$. Thus T has a fixed point. In fact, in this case, the unique fixed point of T is $u = 0$.

4. Results in rectangular b-metric spaces

Our next result ensures the existence of fixed point for interpolative Reich-Rus-Ćirić type contraction in the setting of a rectangular b-metric space.

THEOREM 4.1. *Let (U, h, s) be a complete rectangular b-metric space. If $T : U \to U$ is an interpolative Reich-Rus-Ćirić type contraction, then T has a fixed point in U.*

Proof. Let $u_0 \in U$ and $\{u_n\}$ be defined by $u_n = T^n x_0$ for all $n \in \mathbb{N}$. If there exists $n_0 \in \mathbb{N}$ such that $u_{n_0} = u_{n_0+1}$, then u_{n_0} is clearly a fixed point of T and the proof is complete. So assume that $u_n \neq u_{n+1}$ for all $n \geq 0$. Similarly to the proof of the Theorem 3.5 we get

$$h(u_n, u_{n+1}) \leq \mu h(u_{n-1}, u_n) \text{ for all } n \geq 1, \tag{11}$$

where $\mu = \lambda^{\frac{1}{1-A}} \in [0,1)$. Then $u_n \neq u_{n+k}$ for $n \geq 0, k \geq 1$. Namely, if $u_n = u_{n+k}$ for some $n \geq 0$ and $k \geq 1$ we have that $u_{n+1} = u_{n+k+1}$. Then (11) implies $h(u_{n+1}, u_n) = h(u_{n+k+1}, u_{n+k}) \leq \mu^k h(u_{n+1}, u_n) < h(u_{n+1}, u_n)$, a contradiction. From condition (7) we obtain

$$h(u_m, u_n) \leq \lambda[h(u_{m-1}, u_{n-1})]^B [h(u_{m-1}, u_m)]^A [h(u_{n-1}, u_n)]^{1-A-B}, \tag{12}$$

for all $m, n \in \mathbb{N}$. Using classical inequality

$$u_1^{\lambda_1} u_2^{\lambda_2} u_3^{1-\lambda_1-\lambda_2} \leq \lambda_1 u_1 + \lambda_2 u_2 + (1-\lambda_1-\lambda_2)u_3,$$

for all $u_1, u_2, u_3 \in [0,\infty)$ and $\lambda_1, \lambda_2 \in [0,1), \lambda_1 + \lambda_2 \leq 1$, from (12) we have

$$h(u_m, u_n) \leq \lambda[Bh(u_{m-1}, u_{n-1}) + Ah(u_{m-1}, u_m) + (1-A-B)h(u_{n-1}, u_n)],$$

for all $m, n \in \mathbb{N}$. Therefore, because of (11), we obtain

$$\begin{aligned} h(u_m, u_n) \leq & \lambda[Bh(u_{m-1}, u_{n-1}) + A\lambda^{m-1}h(u_0, u_1) + (1-A-B)\lambda^{n-1}d(u_0, u_1)], \\ \leq & \lambda h(u_{m-1}, u_{n-1}) + A\lambda^{mm}h(u_0, u_1) + (1-A-B)\lambda^n h(u_0, u_1)], \end{aligned}$$

for all $m, n \in \mathbb{N}$. So, from Lemma 2.4 we obtain that $\{u_n\}$ is a Cauchy sequence. By completeness of (U, h, s) there exists $u^* \in U$ such that $\lim_{n\to\infty} u_n = u^*$. Now we obtain that u^* is the fixed point of T. Namely, for any $n \in \mathbb{N}$ we have

$$\begin{aligned} h(u^*, Tu^*) &\leq s[h(u^*, u_{n+1}) + h(u_{n+1}, u_{n+2}) + h(u_{n+2}, Tu^*)] \\ &\leq s[h(u^*, u_{n+1}) + h(u_{n+1}, u_{n+2})] + \\ &\quad s[h(u_{n+1}, u^*)]^B [h(u_{n+1}, u_{n+2})]^a [h(u^*, Tu^*)]^{1-A-B} \to 0 \text{ as } n \to \infty. \end{aligned}$$

So, $Tu^* = u^*$. □

Example 4.2. Let $U = A \cup [1,2]$, where $A = \{0, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}\}$ and the metric $h : U \times U \to [0,\infty)$ be defined as

$$h\left(0, \frac{1}{2}\right) = h\left(\frac{1}{3}, \frac{1}{4}\right) = h\left(\frac{1}{5}, \frac{1}{6}\right) = 0.09, \quad h\left(0, \frac{1}{3}\right) = h\left(\frac{1}{2}, \frac{1}{5}\right) = h\left(\frac{1}{4}, \frac{1}{5}\right) = 0.04,$$

$$h\left(0, \frac{1}{4}\right) = h\left(\frac{1}{2}, \frac{1}{3}\right) = h\left(\frac{1}{4}, \frac{1}{6}\right) = 0.16, \quad h\left(0, \frac{1}{5}\right) = h\left(\frac{1}{2}, \frac{1}{6}\right) = h\left(\frac{1}{3}, \frac{1}{6}\right) = 0.25,$$

$$h\left(0, \frac{1}{6}\right) = h\left(\frac{1}{2}, \frac{1}{4}\right) = h\left(\frac{1}{3}, \frac{1}{5}\right) = 0.36,$$

$$h(u,u) = 0, h(u,v) = h(v,u) \quad \text{for all } u, v \in U, \quad \text{and}$$

$$h(u,v) = (u-v)^2 \quad \text{if } \{u,v\} \cap [1,2] \neq \emptyset.$$

Then (U, h) is a complete rectangular b-metric space with $s = 3$. Define $T : U \to U$ as

$$Tu = \begin{cases} \frac{1}{6}, & \text{if } u \in [1,2], \\ \frac{1}{4}, & \text{if } u \in A \setminus \{\frac{1}{3}\}, \\ \frac{1}{5}, & \text{if } u = \frac{1}{3}. \end{cases}$$

Now if we choose $\lambda = \frac{1}{10}, A = \frac{1}{2}, B = \frac{1}{3}$, then all the conditions of Theorem 4.1 are satisfied and hence T must have a fixed point. Here, the unique fixed point of T is $u = \frac{1}{4}$.

Study of uniqueness of fixed points for the maps we have discussed would be an interesting future work.

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