MATEMATIČKI VESNIK
МАТЕМАТИЧКИ ВЕСНИК
Corrected proof
Available online 21.08.2024

research paper
оригинални научни рад
DOI: 10.57016/MV-h1sbhj29

RULED SURFACES WITH CONSTANT SLOPE DIRECTION IN GALILEAN 3-SPACE

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Abstract. The present study aims to investigate a family of ruled surfaces that are generated by a constant slope direction vector, following the rectifying and normal planes of a given base curve in Galilean 3-space. By examining the properties of this class of ruled surfaces, a number of important results have been obtained, particularly in the case of special base curves. To further illustrate the obtained results, several examples have been provided as applications, and the constructed surfaces have been graphed.

1. Introduction

All physical laws are defined in a frame of observation (frame of reference). These frames enable the definition of physical quantities such as position, velocity, electric field, etc. Galileo established that all laws of motion apply in an inertial frame. He used this principle for the first time in 1632 in his book "Dialogue Concerning the Two Chief World Systems" and explained it using the example of a ship. This principle forms the basis of Newton's laws of motion and Einstein's special theory of relativity. Galilean space has therefore attracted the attention of many authors who study the properties of different surfaces in Galilean space [3, 5–8].

A ruled surface is defined by the property that at least one straight line passing through every point of the surface also lies on the surface. The parametric representation of the ruled surface is $X(s,u) = \alpha(s) + uX(s)$, where the curve α is called the directrix or base curve and $X(s)$ is called the director curve. The straight lines are called the rulings or generators of the ruled surface, while X is called a ruled patch. Ruled surfaces are often studied in differential geometry as they have many interesting geometric properties [1, 9–11, 13].

In [12, Section 2.3] the direction vectors of the generators of the curve α are defined by a fixed angle with respect to the osculating plane u vector. We have extended the

2020 Mathematics Subject Classification: 14H50, 14J26, 53A35
Keywords and phrases: Ruled surface; constant slope; striction curve; developable surface; minimal surface.

ideas presented in [12, Section 2.3] from Euclidean space to Galilean space. We have generalized the concept of ruled surfaces generated by a constant slope direction vector by defining them according to the normal and rectifying planes of the base curve. This generalization has allowed us to obtain new results and insights into the properties of ruled surfaces in Galilean space.

The aim of this paper is to construct ruled surface with constant slope in Galilean 3-space. First, we choose constant slope direction vectors according to normal and rectifying planes of the base curve. Second, we determine the first fundamental form, the second fundamental form, the Gaussian and the mean curvature of ruled surfaces. Then, we investigate the singularity, the developable and minimal conditions of these surfaces based on these invariant properties. Moreover, we give some results by choosing special curves for the base curves of the surfaces. Finally, by choosing an anti-Salkowski curve in Galilean space, we calculate the ruled surfaces based on this curve and present their visuals.

2. Preliminaries

The Galilean metric is defined as

$$\langle x, y\rangle = \begin{cases} x_1y_1, & x_1 \neq 0 \text{ or } y_1 \neq 0 \\ x_2y_2 + x_3y_3, & x_1 = 0 = y_1, \end{cases}$$

where x_i and y_i $(i = 1, 2, 3)$ coordinates of the vectors x, y. If $x = (x_1, x_2, x_3)$ and $y = (y_1, y_2, y_3)$ (at least one vector is non-isotropic) are vectors in Galilean space, we define the vector product of x and y as follows [14]:

$$x \times y = \begin{vmatrix} 0 & e_2 & e_3 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix}, \quad x_1 \neq 0 \text{ or } y_1 \neq 0.$$

A Euclidean rotation around the non-isotropic x-axis is given by

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

where θ is the Euclidean angle [7]. An inertial reference frame is defined as a coordinate system that moves with a constant velocity. The Galilean transformation between two inertial frames (x, y, z) and (x', y', z') is defined as

$$\begin{aligned} x' &= a + x, \\ y' &= b + cx + (\cos\theta)\, y + (\sin\theta)\, z, \\ z' &= d + ex - (\sin\theta)\, y + (\cos\theta)\, z, \end{aligned}$$

where a, b, c, d, e, and θ are some constants. In Galilean space, since two inertial frames are related by a Galilean transformation, all physical laws are the same in all inertial reference frames.

Let $\alpha : I \subset \mathbb{R} \to \mathbb{G}_3$ be a unit speed curve with $\alpha(s) = (s, y(s), z(s))$. The Frenet frame is defined by $\{T(s)=\alpha'(s), N(s), B(s)\}$ for the curve $\alpha(s)$ in Galilean $3-$ space. The Frenet equations are given by

$$T'(s) = \kappa(s) N(s),$$
$$N'(s) = \tau(s) B(s),$$
$$B'(s) = -\tau(s) N(s),$$

with the curvature $\kappa(s) = \|\alpha''(s)\|$ and the torsion $\tau(s) = \frac{1}{\kappa^2(s)} \det(\alpha', \alpha'', \alpha''')$ (see [2]).

Let $X(u,v) = (x(u,v), y(u,v), z(u,v))$ be a parametric surface in the Galilean 3-space. The intrinsic geometry of the parametric surface $X(u,v)$ at the point $X(u_0,v_0)$ is obtained by the first fundamental form. The first fundamental form of a surface is $I = (g_1 du + g_2 dv)^2 + \varepsilon(h_{uu}(du)^2 + 2h_{uv} du dv + h_{vv}(dv)^2)$, where $g_1 := x_u = \frac{\partial x}{\partial u}$, $g_2 := x_v$, $h_{uv} := y_u y_v + z_u z_v$, $h_{uu} := y_u^2 + z_u^2$, $h_{vv} := y_v^2 + z_v^2$, and

$$\varepsilon = \begin{cases} 0, & \text{if the direction } du : dv \text{ is non- isotropic,} \\ 1, & \text{if the direction } du : dv \text{ is isotropic.} \end{cases}$$

The Gauss map of the surface $X(u,v)$ is defined as

$$U = \frac{1}{W}(X_s \times X_u) = \frac{1}{W}(0, -x_u z_v + x_v z_u, x_u y_v - x_v y_u)$$

where $W = \sqrt{(x_u z_v - x_v z_u)^2 + (x_v y_u - x_u y_v)^2}$. The second fundamental form is given by $II = L_{11}(du)^2 + 2L_{12} du dv + L_{22}(dv)^2$, where

$$L_{ij} = \frac{1}{g_1} \langle g_1 (0, y_{,ij}, z_{,ij}) + g_{i,j} (0, y_u, z_u), U \rangle \quad \text{for } g_1 \neq 0$$

or

$$L_{ij} = \frac{1}{g_2} \langle g_2 (0, y_{,ij}, z_{,ij}) + g_{i,j} (0, y_v, z_v), U \rangle \quad \text{for} \quad g_2 \neq 0$$

with $y_{,ij} = \frac{\partial y}{\partial u_i u_j}$, $j = 1,2$ and $u_1 := u$, $u_2 := v$. The Gaussian and mean curvatures K and H of the surface are calculated as follows:

$$K := \frac{L_{11}L_{22} - L_{12}^2}{W^2} \quad \text{and} \quad H := \frac{g_2^2 L_{11} - 2g_1 g_2 L_{12} + g_1^2 L_{22}}{2W^2}. \tag{1}$$

A surface in Galilean 3-space is called flat (resp. minimal) surface if its Gaussian (resp. mean) curvature is zero [4, 15]. The principal curvatures k_1 and k_2 of the surface X are given as

$$k_1 = 2H \quad \text{and} \quad k_2 = \frac{L_{11}L_{22} - L_{12}^2}{g_2^2 L_{11} - 2g_1 g_2 L_{12} + g_1^2 L_{22}}. \tag{2}$$

3. Ruled Surface with constant slope in Galilean 3-space

In this section, we will investigate the characterizations of the ruled surfaces whose director vector forms a constant angle with the normal and rectifying plane of the base curve α. We have presented the necessary conditions for these surfaces to be developable and minimal.

3.1 Ruled surface with constant slope with respect to the normal plane

In this subsection, we give characterizations of ruled surface formed by a direction vector with constant slope with respect to the normal plane of the curve α. The ruled surface around the curve α is defined by $\mathcal{X}(s,u)=\alpha(s)+uF(s)$ with the generator vector $F(s)=\cos\theta(s)N(s)+\sin\theta(s)B(s)+\sigma T(s)$ where σ is an arbitrary constant. The normal vector field of the surface $\mathcal{X}(s,u)$ is

$$\begin{aligned}\mathcal{U}=&\frac{1}{W}\{[-\sin\theta(s)+u\sigma(\theta'(s)+\tau(s))\cos\theta(s)]\,N(s)\\&+[\cos\theta(s)-u\sigma(-(\theta'(s)+\tau(s))\sin\theta(s)+\sigma\kappa(s))]\,B(s)\},\end{aligned}$$

where $W=\|X_s\times X_u\|$. The coefficients of the second fundamental form of $\mathcal{X}(s,u)$ are found with $g_1=1$ and $g_2=\sigma$,

$$\begin{aligned}L_{11}=&\frac{1}{W}\{(-\sin\theta+u\sigma\mu)(\kappa+u\lambda'-u\mu\tau)+(u\lambda\tau+u\mu')(\cos\theta-u\sigma\lambda)\}\\L_{12}=&\frac{1}{W}\left(-\lambda\sin\theta+\mu\cos\theta\right),\quad\text{and}\quad L_{22}=0,\end{aligned}$$

where $\lambda:=-(\theta'(s)+\tau(s))\sin\theta(s)+\sigma\kappa(s)$, and $\mu:=(\theta'(s)+\tau(s))\cos\theta(s)$.

THEOREM 3.1. *The ruled surface $\mathcal{X}(s,u)$ is developable if and only if*

$$\theta'(s)+\tau(s)=\sigma\kappa(s)\sin\theta(s).$$

Proof. The Gauss curvature of the ruled surface $\mathcal{X}(s,u)$ is calculated from (2) as $K=-\frac{L_{12}^2}{W^2}$. The surface $\mathcal{X}(s,u)$ is a developable surface if and only if $-\lambda\sin\theta+\mu\cos\theta=0$. We therefore obtain the differential equation $\theta'(s)+\tau(s)=\sigma\kappa(s)\sin\theta(s)$. □

COROLLARY 3.2. *Let θ be a constant. The ruled surface $\mathcal{X}(s,u)$ is developable if and only if the base curve α is a general helix with $\frac{\tau(s)}{\kappa(s)}=\sigma\sin\theta$.*

Proof. The ruled surface $\mathcal{X}(s,u)$ is developable if and only if the Gauss curvature $K=-\frac{1}{W^4}(\theta'(s)+\tau(s)-\sigma\kappa(s)\sin\theta(s))^2$ is zero. If the parameter θ is a constant, then we get $\tau(s)-\sigma\kappa(s)\sin\theta=0$. The curve α is therefore a general helix. □

COROLLARY 3.3. *Let the base curve α of the surface $\mathcal{X}(s,u)$ be a plane curve in Galilean 3-space. The ruled surface $\mathcal{X}(s,u)$ is developable if and only if the condition $\theta(s)=2\cot^{-1}(e^{c-\sigma\int\kappa(s)ds})$ is satisfied .*

Proof. Let α be a planar curve in Galilean 3-space. The surface $\mathcal{X}(s,u)$ is developable if and only if the differential equation $\theta'(s)=\sigma\kappa(s)\sin\theta(s)$ is satisfied. From the solution of the last differential equation, we obtain $\theta(s)=2\cot^{-1}(e^{c-\sigma\int\kappa(s)ds})$ where c is an arbitrary constant. □

COROLLARY 3.4. *Let the base curve α be a circular helix ($\kappa(s)=const$ and $\tau(s)=const$) in Galilean 3-space. The ruled surface $\mathcal{X}(s,u)$ is developable if and only if*

$$\theta(s)=2\tan^{-1}\left(\frac{1}{\tau}\left(\sigma\kappa-\sqrt{\tau^2-(\sigma\kappa)^2}\tan\left(\frac{\sqrt{\tau^2-(\sigma\kappa)^2}(k_1+s)}{2}\right)\right)\right),$$

where k_1 is an arbitrary constant.

Proof. Let the curve α be a circular helix, i.e. $\kappa(s) = const$ and $\tau(s) = const$ in Galilean 3-space. If $\mathcal{X}(s,u)$ is developable if and only if the differential equation $\theta'(s) + \tau = \sigma\kappa\sin\theta(s)$ is satisfied. From the solution of the last differential equation, we obtain the desired result for the parameter $\theta(s)$. □

COROLLARY 3.5. *Let the base curve α be a general helix ($\tau(s) = m\kappa(s)$, $m = const$) in Galilean 3-space. The ruled surface $\mathcal{X}(s,u)$ is developable if and only if*

$$\theta(s) = 2\tan^{-1}\left(\frac{1}{m}\left(\sigma - \sqrt{m^2-\sigma^2}\tan\left(\frac{\sqrt{m^2-\sigma^2}(c_1 + \int\kappa(s)ds)}{2}\right)\right)\right),$$

where c_1 is an arbitrary constant.

THEOREM 3.6. *The ruled surface $\mathcal{X}(s,u)$ is a minimal surface if and only if*

$$\begin{aligned}&\{\sigma(-\sin\theta + u\sigma\mu)(\kappa + u\lambda') + \sigma u\mu'(\cos\theta - u\sigma\lambda)\\ &+u\sigma^2\kappa\tau\cos\theta - u^2\sigma^2\tau(\lambda^2+\mu^2) - 2(\theta' + \tau - \sigma\kappa\sin\theta)\} = 0.\end{aligned} \tag{3}$$

Proof. The mean curvature of $\mathcal{X}(s,u)$ is calculated as follows

$$\begin{aligned}H = \frac{\sigma}{2W^3}&\{\sigma(-\sin\theta + u\sigma\mu)(\kappa + u\lambda') + \sigma u\mu'(\cos\theta - u\sigma\lambda)\\ &+u\sigma^2\kappa\tau\cos\theta - u^2\sigma^2\tau(\lambda^2+\mu^2) - 2(\theta' + \tau - \sigma\kappa\sin\theta)\}.\end{aligned}$$

The surface $\mathcal{X}(s,u)$ is therefore a minimal, i.e. $H = 0$, if and only if the condition (3) is satisfied. □

COROLLARY 3.7. *Let $X(u,v)$ be a surface with a planar base curve and θ constant. If $\kappa'(s) \neq 0$, then it has minimal points along the curve $u = \frac{\kappa(s)}{\sigma\kappa\prime(s)}$.*

Proof. Since the parameter θ is a constant and $\tau(s) = 0$, then we obtain that the mean curvature of $\mathcal{X}(s,u)$ is equal to $H = \dfrac{\sigma^2\sin\theta}{2(1 - 2u\sigma^2\kappa\cos\theta + u^2\sigma^4\kappa^2)^{3/2}}(\kappa(s) - u\sigma\kappa'(s))$. The surface is a minimal surface if and only if $H = 0$. Therefore, $\kappa(s) - u\sigma\kappa'(s) = 0$ is satisfied. It follows from the last equation that $u = \frac{\kappa(s)}{\sigma\kappa'(s)}$ for $\kappa'(s) \neq 0$. □

If there is a common perpendicular to two constructive rulings in the ruled surface, then the foot of the common perpendicular to the main rulings is called the central point. The locus of the central point is called the striction curve.

PROPOSITION 3.8. *The striction curve of the ruled surface $\mathcal{X}(s,u)$ is*

$$\beta(s) = \alpha(s) - \frac{\sigma}{(\theta'(s) + \tau(s) - \sigma\kappa(s)\sin\theta(s))^2 + \sigma^2\kappa^2(s)\cos^2\theta(s)}F(s).$$

Proof. The striction curve of the ruled surface can be of two types depending on whether $F'(s)$ is an isotropic or a non-isotropic vector in Galilean space. The striction curve of the ruled surface is calculated according to the following formula

$$\beta(s) = \alpha(s) - \frac{\langle T(s), F(s)\rangle}{\langle F'(s), F'(s)\rangle}F(s).$$

Since the derivative of the generator vector F with respect to s is an isotropic vector, then we get the desired result. □

3.2 Ruled surface with constant slope with respect to rectifying plane

In this subsection, we choose a direction vector G that forms a constant angle with the isotropic rectifying plane of the base curve. Using this vector and the $\alpha(s)$ generating curve, we have defined the ruled surface $\mathbf{X}(s,v)=\alpha(s)+vG(s)$ with $G(s)=\zeta N(s)+\theta(s)B(s)$, where ζ is an arbitrary constant. By choosing the direction vector G in this way, we have obtained a family of ruled surfaces that have interesting properties such as constant mean curvature and minimal surface character. The first and second partial derivatives of the surface $\mathbf{X}(s,v)$ are calculated as follows:

$$\mathbf{X}_s = T(s)+(-v\theta(s)\tau(s))N(s)+v(\zeta\tau(s)+\theta'(s))B(s),$$
$$\mathbf{X}_v = \zeta N(s)+\theta(s)B(s),$$

and

$$\begin{aligned}\mathbf{X}_{ss} &= [\kappa(s)-v(\theta'(s)\tau(s)+\theta(s)\tau'(s))-v\tau(s)(\zeta\tau(s)+\theta'(s))]\,N(s)\\ &\quad+[v(-\theta(s)\tau)(s)+\zeta\tau'(s)+\theta''(s)]\,B(s),\\ \mathbf{X}_{sv} &= -\theta(s)\tau(s)N(s)+(\zeta\tau(s)+\theta'(s))B(s),\\ \mathbf{X}_{vv} &= 0.\end{aligned}$$

Using the above equations, the normal vector field of the surface $\mathbf{X}(s,v)$ is obtained as follows:

$$\mathbf{U}=\frac{1}{\sqrt{\theta^2(s)+\zeta^2}}\left[-\theta(s)N(s)+\zeta B(s)\right].$$

Equation (1) gives the coefficients of the first and second fundamental forms of $\mathbf{X}(s,v)$ are calculated as

$$g_1=1 \quad\text{and}\quad g_2=0,$$
$$\begin{aligned}L_{11} &= \frac{1}{\sqrt{\theta^2(s)+\zeta^2}}\{-\theta(s)\left[\kappa(s)-v(\theta'(s)\tau(s)+\theta(s)\tau'(s))-v\tau(s)(\zeta\tau(s)+\theta'(s))\right]\\ &\qquad+v\left[-\theta(s)\tau(s)+\zeta\tau'(s)+\theta''(s)\right]\},\\ L_{12} &= \frac{1}{\sqrt{\theta^2(s)+\zeta^2}}\left\{\tau(s)\left(\theta^2(s)+\zeta^2\right)+\zeta\theta'(s)\right\}, \quad\text{and}\quad L_{22}=0.\end{aligned}$$

Theorem 3.9. *The ruled surface $\mathbf{X}(s,v)$ is developable if and only if*

$$\theta(s)=-\tan\left(\zeta\int\tau(s)ds\right)+c, \quad c=const.$$

Proof. The Gauss curvature of the surface $\mathbf{X}(s,v)$ is

$$K=-\frac{1}{\left(\theta^2(s)+\zeta^2\right)^2}\left\{\theta(s)\left(\theta^2(s)+\zeta^2\right)+\zeta\theta'(s)\right\}^2.$$

To show that the ruled surface $\mathbf{X}(s,v)$ is developable, we must show that it has zero Gauss curvature. If we require that the Gauss curvature is zero, we obtain the following ordinary differential equation $\tau(s)\left(\theta^2(s)+\zeta^2\right)+\zeta\theta'(s)=0.$, whose solution is given as follows $\theta(s)=-\tan\left(\zeta\int\tau(s)ds\right)+c$, where c is an arbitrary constant. □

Theorem 3.10. *The surface $\mathbf{X}(s,v)$ is always a minimal surface.*

Proof. The equation (2) shows that the mean curvature of the ruled surface $\mathbf{X}(s,v)$ is zero ($H=0$). From this we can conclude that the surface $\mathbf{X}(s,v)$ is always a minimal surface. □

4. Example

Let $\beta=\beta(s)$ be an admissible unit speed curve with the parameterization

$$\beta(s)=\Bigg(s,\quad \frac{16}{289}\left(8\sin(s)\sinh\left(\frac{s}{4}\right)-15\cos(s)\cosh\left(\frac{s}{4}\right)\right), \\ -\frac{16}{289}\left(8\cos(s)\sinh\left(\frac{s}{4}\right)+15\sin(s)\cosh\left(\frac{s}{4}\right)\right)\Bigg),$$

with the Frenet frame apparatus

$$\begin{aligned} T(s)&=\Bigg(1,\frac{4}{17}\left(4\sin(s)\cosh\left(\frac{s}{4}\right)+\cos(s)\sinh\left(\frac{s}{4}\right)\right), \\ &\quad -\frac{4}{17}\left(4\cos(s)\cosh\left(\frac{s}{4}\right)-\sin(s)\sinh\left(\frac{s}{4}\right)\right)\Bigg), \\ N(s)&=(0,\cos(s),\sin(s))\,, \\ B(s)&=(0,-\sin(s),\cos(s))\,, \\ \kappa(s)&=\cosh\left(\frac{s}{4}\right), \qquad \tau(s)=1. \end{aligned}$$

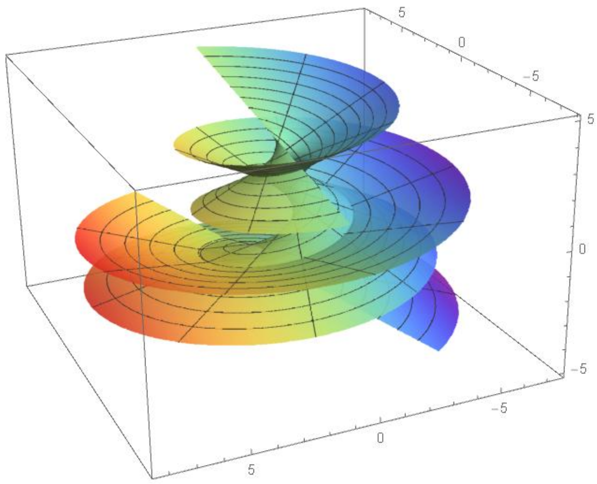

Figure 1: The ruled surface $\mathcal{X}(s,u)=\alpha(s)+uF(s)$ with constant slope direction vector with respect to normal plane of the base curve $\alpha(s)$ where $F(s)=(\cos s)\,N(s)+(\sin s)\,B(s)+\frac{1}{3}T(s)$ and $s\in[-3,3]$, $u\in[-6,6]$.

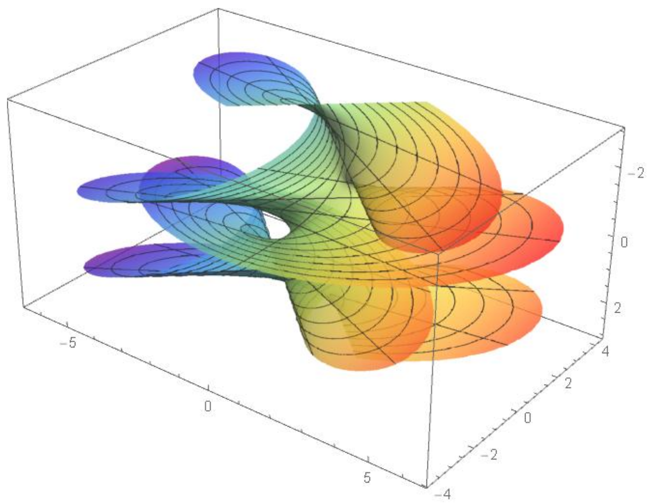

Figure 2: The minimal surface $\mathbf{X}(s,v)=\alpha(s)+vG(s)$ with constant slope vector with respect to rectifying plane of the curve $\alpha(s)$ where $G(s)=\frac{1}{3}N(s)+(\sin 2s)\,B(s)$ and $s\in[-3,3]$, $v\in[-6,6]$.

In Theorem 3.9, if we choose $\zeta(s)=-1/4$, then the function $\theta(s)$ can be calculated as $\theta(s)=\tan(s/4)+c$. When these values are substituted into the surface $\mathbf{X}(s,v)=\alpha(s)+vG(s)$, the resulting developable surface $\mathbf{X}$ can be seen in Figure 3. Additionally, this surface is minimal.

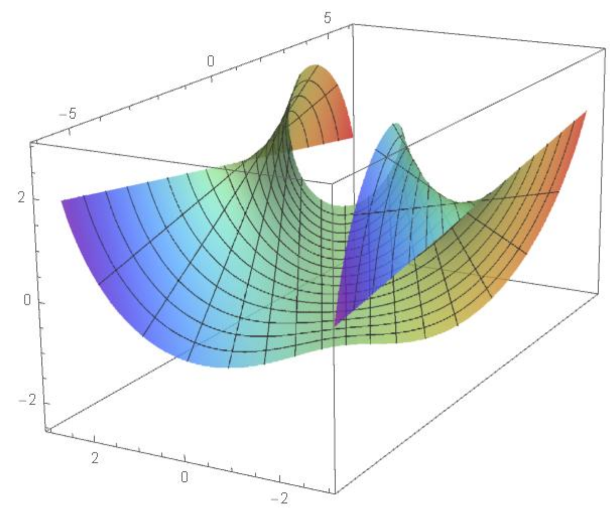

Figure 3: The developable surface $\mathbf{X}(s,v)$ with $s\in[-3,3], v\in[-6,6]$.

5. Conclusions

In this paper, we study the differential geometry of ruled surfaces in Galilean 3-space generated by a constant slope direction vector relative to the rectifying and normal planes of the base curve. We calculate the first and second quadratic forms of these surfaces and analyze the conditions for them to be both minimal and developable.

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(received 10.03.2022; in revised form 06.09.2023; available online 21.08.2024)

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