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A NOTE ON BOUNDARY VALUES FOR THE POISSON TRANSFORM

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Abstract. We determine boundary values in distributional and pointwise sense for the Poisson transform of a certain class of weighted distributions.

1. Introduction

The authors of [1] have proved that the weighted space $w^{n+1}\mathcal{D}'_{L^1}$ is the optimal space of tempered distributions admitting $\mathcal{S}'$-convolution with the euclidean version of the Poisson kernel P_y. Moreover, they have studied in [2] the boundary behavior of the $\mathcal{S}'$-convolution $T * P_y$ when $T \in w^{n+1}\mathcal{D}'_{L^1}$, obtaining in this way harmonic extensions of weighted integrable distributions to the upper half-space, where the convergence is understood in the sense of the strong topology of the space.

The goal of this note is to study the boundary values of $T * P_y$ when T is a function in the weighted space $L^p\left(w^{-n-1}\right)$, $1 \le p < \infty$, or a weighted distribution in an intermediate space between $L^p\left(w^{-n-1}\right)$ and $w^{n+1}\mathcal{D}'_{L^1}$. Since $L^p\left(w^{-n-1}\right) \subset L^1\left(w^{-n-1}\right) \subset w^{n+1}\mathcal{D}'_{L^1}$, these considerations make sense.

First, we will introduce briefly some definitions and results that will be required along this work, trying to avoid technicalities that could be tedious the reading of the prerequisites.

With B we indicate the space of smooth functions $\varphi : \mathbf{R}^n \to \mathbf{C}$ such that $\partial^\alpha\varphi$ is bounded in $\mathbf{R}^n$ for each multi-index α. We consider in B the topology of the uniform convergence in $\mathbf{R}^n$ of each derivative. With $\dot{B}$ we indicate the closed subspace of B that consists of those smooth functions $\varphi : \mathbf{R}^n \to \mathbf{C}$ such that $\partial^\alpha\varphi \to 0$ as $|x| \to \infty$, for each multi-index α. The space C_0^∞ is dense in $\dot{B}$ but not in B.

The space $\mathcal{D}'_{L^1}$ of integrable distributions is, by definition, the topological dual of the space $\dot{B}$, endowed with the strong dual topology.

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Since C_0^∞ is dense in $\dot{B}$, the space $\mathcal{D}'_{L^1}$ is a subspace of $\mathcal{D}'$. According to [5], each distribution $T \in \mathcal{D}'_{L^1}$ can be represented as

$$T = \sum\nolimits_{finite} \partial^\alpha f_\alpha \tag{1}$$

where $f_\alpha \in L^1$.

We can also consider an alternative topology in the space B. We will denote with B_c the space B endowed with the following notion of convergence: a sequence $\{\varphi_j\}$ converges to φ in B_c if for each multi-index α, $\sup_j \|\partial^\alpha \varphi_j\|_\infty < \infty$ and the sequence $\{\partial^\alpha \varphi_j\}$ converges to $\partial^\alpha \varphi$ uniformly on compact sets of $\mathbf{R}^n$. It can be proved that C_0^∞, and so $\dot{B}$, is dense in B_c and also that the space $\mathcal{D}'_{L^1}$ is the topological dual of B_c.

Now we consider the notion of $\mathcal{S}'$-convolution whose aim is to preserve the Fourier exchange formula.

Definition 1. [6] Given two tempered distributions T and S, it is said that they are $\mathcal{S}'$-convolvable if $T(\check{S} * \varphi) \in \mathcal{D}'_{L^1}$ for every $\varphi \in \mathcal{S}$. If this is the case, the map

$$\begin{aligned} &\mathcal{S} \to \mathbf{C} \\ &\varphi \mapsto (T(\check{S} * \varphi), 1)_{\mathcal{D}'_{L^1}, B_c} \end{aligned} \tag{2}$$

is linear and continuous. Thus, it defines a tempered distribution which will be denoted by $T * S$.

This operation coincides with the classical convolution defined by L. Schwartz, in all the cases in which both make sense.

In the paper [1] the problem of finding the optimal spaces of tempered distributions that are $\mathcal{S}'$-convolvable with the Poisson kernel is solved, for the Euclidean version

$$P_y(x) = \frac{c_n}{y^n} \frac{1}{\left(\frac{|x|^2}{y^2} + 1\right)^{\frac{n+1}{2}}}, \tag{3}$$

where $c_n = \Gamma\left(\frac{n+1}{2}\right)/\pi^{\frac{n+1}{2}}$, $y > 0$.

The authors of [1] identified distributions in appropriate weighted spaces, as those $\mathcal{S}'$-convolvable with the euclidean version of the Poisson kernel. These distributions are defined as follows.

Definition 2. Let $w(x) = \left(1 + |x|^2\right)^{\frac{1}{2}}$, for $x \in \mathbf{R}^n$. Then, given $\mu \in \mathbf{R}$ we consider

$$w^\mu \mathcal{D}'_{L^1} = \left\{T \in \mathcal{S}' : w^{-\mu} T \in \mathcal{D}'_{L^1}\right\}$$

with the topology induced by the map

$$\begin{aligned} w^\mu \mathcal{D}'_{L^1} &\to \mathcal{D}'_{L^1} \\ T &\mapsto w^{-\mu} T. \end{aligned}$$

The space $w^{\mu}\mathcal{D}'_{L^1}$ is the topological dual of the spaces $w^{-\mu}\dot{B}$ and $w^{-\mu}B_c$.

Theorem 3. [1] *Given $T \in \mathcal{S}'$, the following statements are equivalent:*

1. $T \in w^{n+1}\mathcal{D}'_{L^1}$.
2. T *is $\mathcal{S}'$-convolvable with P_y, for each $y > 0$.*

Given $T \in w^{n+1}\mathcal{D}'_{L^1}$, it was proved in [1] that the $\mathcal{S}'$-convolution $T * P_y$ is a function defined on $\mathbf{R}^n$ as

$$(T * P_y)(x) = \left(w^{-n-1}(t)T_t, w^{n+1}(t)P_y(x-t)\right)_{\mathcal{D}'_{L^1}, B_c}. \tag{4}$$

Also, in [2] it is shown that

$$w^{n+1}\mathcal{D}'_{L^1} = \left\{T \in \mathcal{S}' : T = \sum_{finite} \partial^{\alpha} f_{\alpha},\ f_{\alpha} \in L^1\left(w^{-n-1}\right)\right\}. \tag{5}$$

2. Main results

According to [2], when $T \in w^{n+1}\mathcal{D}'_{L^1}$ the $\mathcal{S}'$-convolution $T * P_y$ is a function in $L^1\left(w^{-n-1}\right)$. Thus, we can consider the family of operators $\Lambda_y : L^p\left(w^{-n-1}\right) \to L^1\left(w^{-n-1}\right)$, $1 \le p < \infty$, such that $\Lambda_y(f) = P_y * f$.

Lemma 4. *For each $y > 0$, Λ_y is a bounded operator from $L^p\left(w^{-n-1}\right)$ into itself, $1 \le p < \infty$, and moreover $\|\Lambda_y\| \le C_{n,p}(1+y)^{1/p}$.*

Proof. Applying Jensen inequality and the semigroup property of the family $(P_\eta)_{\eta>0}$ we have

$$\begin{aligned}
\|P_y * f\|^p_{L^p(w^{-n-1})} &\le \int_{\mathbf{R}^n}\int_{\mathbf{R}^n} |f(t)|^p\, P_y(x-t)\, dt\, \frac{dx}{\left(1+|x|^2\right)^{\frac{n+1}{2}}} \\
&= C_n \int_{\mathbf{R}^n} |f(t)|^p\, (P_y * P_1)(t)\, dt \\
&= C_n \int_{\mathbf{R}^n} |f(t)|^p\, (P_{1+y})(t)\, dt \\
&\le C_n (1+y) \int_{\mathbf{R}^n} |f(t)|^p\, \frac{dt}{\left(1+|t|^2\right)^{\frac{n+1}{2}}} \\
&= C_n (1+y)\, \|f\|^p_{L^p(w^{-n-1})}.
\end{aligned}$$

This concludes the proof. ■

It has been proved in [2, Th. 3.6] that for $f \in L^1\left(w^{-n-1}\right)$ the $\mathcal{S}'$-convolution $P_y * f \to f$ in $L^1\left(w^{-n-1}\right)$ as $y \to 0^+$. This is also true for $1 \le p < \infty$ as we prove below.

Lemma 5. *For each* $f \in L^p\left(w^{-n-1}\right)$, $1 \le p < \infty$, $P_y * f \to f$ *in* $L^p\left(w^{-n-1}\right)$ *as* $y \to 0^+$.

Proof. When $f \in C_c$ it is clear that $P_y * f \to f$ in L^p, hence in $L^p\left(w^{-n-1}\right)$ as $y \to 0^+$. Now, if $f \in L^p\left(w^{-n-1}\right)$ since C_c is dense in $L^p\left(w^{-n-1}\right)$, given $\varepsilon > 0$ there exists $g \in C_c$ such that $\|f-g\|_{L^p(w^{-n-1})} < \varepsilon$ and also $\|P_y * g - g\|_{L^p(w^{-n-1})} < \varepsilon$. Thus

$$\|P_y * f - f\|_{L^p(w^{-n-1})} \le \|P_y * (f-g)\|_{L^p(w^{-n-1})} + 2\varepsilon. \tag{6}$$

Jensen inequality and the semigroup property of the family $(P_\eta)_{\eta>0}$ imply

$$\|P_y * (f-g)\|^p_{L^p(w^{-n-1})} \le C_n \int_{\mathbf{R}^n} |f-g|^p(x)\, P_{1+y}(x)\, dx. \tag{7}$$

For $0 < y \le 2$ we have that

$$|f-g|^p(x)\, P_{1+y}(x) \le C_n\, |f-g|^p(x)\, P_1(x)$$
$$|f-g|^p(x)\, P_{1+y}(x) \to |f-g|^p(x)\, P_1(x) \text{ as } y \to 0^+$$

and by Lebesgue Dominated Convergence Theorem we obtain that

$$\lim_{y\to 0^+} \int_{\mathbf{R}^n} |f-g|^p(x)\, P_{1+y}(x)\, dx = C_n\, \|f-g\|^p_{L^p(w^{-n-1})}.$$

Thus, choosing $y > 0$ small enough we can conclude from (6) and (7) that

$$\|P_y * f - f\|_{L^p(w^{-n-1})} < 3\varepsilon + C_{n,p}\varepsilon.$$

This concludes the proof. ∎

Remark 6. Lemmas 4 and 5 imply that the family of operators $(\Lambda_y)_{y>0}$ is a strongly continuous semigroup of convolution operators in the Banach space $L^p\left(w^{-n-1}\right)$, $1 \le p < \infty$, such that $\|\Lambda_y\| \le C_{n,p}(1+y)^{1/p}$.

The previous results lead us to consider the following subspace of $w^{n+1}\mathcal{D}'_{L^1}$:

$$\mathcal{A}_p = \left\{T \in \mathcal{S}' : T = \sum_{finite} \partial^\alpha f_\alpha,\ f_\alpha \in L^p\left(w^{-n-1}\right)\right\},\ 1 \le p < \infty.$$

Clearly, $\mathcal{A}_p$ is closed under differentiation and $\partial^\alpha(T * P_y) = (\partial^\alpha T) * P_y$ for each $T \in \mathcal{A}_p$ and each multi-index α.

Now, let $T \in \mathcal{A}_p$. Using the fact that the $\mathcal{S}'$-convolution $T * P_y$ can be computed as

$$\begin{aligned}(T * P_y, \varphi)_{\mathcal{S}',\mathcal{S}} &= ((P_y * \varphi)\, T, 1)_{\mathcal{D}'_{L^1}, B_c} \\ &= (T, P_y * \varphi)_{w^{n+1}\mathcal{D}'_{L^1}, w^{-n-1}B_c}\end{aligned}$$

(see [1]), we can readily see for $T = \sum_{finite} \partial^\alpha f_\alpha$, $f_\alpha \in L^p\left(w^{-n-1}\right)$ that

$$(T * P_y)(x) = \sum_{finite} (-1)^{|\alpha|} \int_{\mathbf{R}^n} f_\alpha(t)\, \partial_t^\alpha P_y(x-t)\, dt. \tag{8}$$

Thus, we can prove the following result.

LEMMA 7. *If $T \in \mathcal{A}_p$, $1 \le p < \infty$, then $T * P_y$ is a smooth function and $\partial^\alpha (T * P_y) \in L^p\left(w^{-n-1}\right)$ for every multi-index α and each $y > 0$.*

Proof. Since $\mathcal{A}_p$ is closed under differentiation, it suffices to prove that $T * P_y \in L^p\left(w^{-n-1}\right)$ for every $T \in \mathcal{A}_p$.

Without loss of generality let us assume that $T = \partial^\alpha f$, $f \in L^p\left(w^{-n-1}\right)$. Then, according to (8)

$$(T * P_y)(x) = (-1)^{|\alpha|} \int_{\mathbf{R}^n} f(t)\, \partial_t^\alpha P_y(x-t)\, dt.$$

Since

$$\begin{aligned}
\left|\partial_t^\alpha P_y(x-t)\right| &\le \frac{C_{n,\alpha}}{y^{n+|\alpha|}} \left(1 + \frac{|x-t|^2}{y^2}\right)^{-\frac{(n+1+|\alpha|)}{2}} \\
&\le \frac{C_{n,\alpha}}{y^{n+|\alpha|}} \left(1 + \frac{|x-t|^2}{y^2}\right)^{-\frac{(n+1)}{2}}
\end{aligned}$$

we have that

$$\begin{aligned}
\left|(T * P_y)(x)\right| &\le \frac{C_{n,\alpha}}{y^{n+|\alpha|}} \int_{\mathbf{R}^n} |f(t)| \left(1 + \frac{|x-t|^2}{y^2}\right)^{-\frac{(n+1)}{2}} dt \\
&= \frac{C_{n,\alpha}}{y^{|\alpha|}} \left(P_y * |f|\right)(x).
\end{aligned}$$

Therefore

$$\begin{aligned}
\|T * P_y\|_{L^p(w^{-n-1})} &\le \frac{C_{n,\alpha}}{y^{|\alpha|}} \left\|P_y * |f|\right\|_{L^p(w^{-n-1})} \\
&\le \frac{C_{n,\alpha,p}}{y^{|\alpha|}} (1+y)^{1/p} \|f\|_{L^p(w^{-n-1})}. \qquad (9)
\end{aligned}$$

This concludes the proof. ■

REMARK 8. For $T \in \mathcal{A}_p$, $1 \le p < \infty$, $T = \sum_{finite} \partial^\alpha f_\alpha$, $f_\alpha \in L^p\left(w^{-n-1}\right)$ we have

$$\|T * P_y\|_{L^p(w^{-n-1})} \le \sum_{finite} \frac{C_{n,\alpha,p}}{y^{|\alpha|}} (1+y)^{1/p} \|f_\alpha\|_{L^p(w^{-n-1})}.$$

We also have a version for the subspace $\mathcal{A}_p$ of [2, Th. 3.6].

THEOREM 9. *Given $T \in \mathcal{A}_p$, $1 \le p < \infty$, the $\mathcal{S}'$-convolution $T * P_y \to T$ as $y \to 0^+$ in $\mathcal{A}_p$.*

Proof. Since the topology in $\mathcal{A}_p$ is the inherited from $w^{n+1}\mathcal{D}'_{L^1}$, $L^p\left(w^{-n-1}\right) \hookrightarrow L^1\left(w^{-n-1}\right) \hookrightarrow w^{n+1}\mathcal{D}'_{L^1}$, and $\mathcal{A}_p$ is closed under differentiation, it suffices to show that $P_y * f \to f$ in $L^p\left(w^{-n-1}\right)$ as $y \to 0^+$. But this last assertion was proved in Lemma 5. ■

Next, we will approach the problem of pointwise convergence of the $\mathcal{S}'$-convolution $P_y * f$ when $y \to 0^+$ and $f \in L^1\left(w^{-n-1}\right)$.

THEOREM 10. *Given $f \in L^1\left(w^{-n-1}\right)$, $(P_y * f)(x) \to f(x)$ as $y \to 0^+$ for every Lebesgue point x of f and therefore, almost everywhere on $\mathbf{R}^n$.*

Proof. For the first part of the proof, we will follow [3, Th. 8.15]. Let x be a Lebesgue point of f. Thus for every $\beta > 0$ there exists $\lambda > 0$ such that

$$\int_{|t|<r} |f(x-t) - f(x)|\, dt \leq C\beta r^n, \quad 0 < r \leq \lambda. \tag{10}$$

Now, we consider the integrals

$$I_1 = \int_{|t|<\lambda} |f(x-t) - f(x)|\, P_y(t)\, dt$$

$$I_2 = \int_{|t|\geq\lambda} |f(x-t) - f(x)|\, P_y(t)\, dt.$$

We will prove that I_1 is bounded by $C\beta$, where C is a constant independent of y, and that $I_2 \to 0$ as $y \to 0^+$. This will imply the desired conclusion.

To get an estimate for I_1, let $y > 0$ fixed.

Select the unique integer N satisfying

$$\begin{cases} 2^N \leq \frac{\lambda}{y} < 2^{N+1} & \text{if } \frac{\lambda}{y} \geq 1 \\ 0 & \text{if } \frac{\lambda}{y} < 1. \end{cases}$$

We decompose the ball $|t| < \lambda$ as the union of the annuli $\frac{\lambda}{2^j} \leq |t| < \frac{\lambda}{2^{j-1}}$, $1 \leq j \leq N$, and the ball $|t| < \frac{\lambda}{2^N}$. By means of (10) and using the estimates

$$|P_y(t)| \leq Cy^{-n}\left[\frac{2^{-j}\lambda}{y}\right]^{-n-1} \text{ on the } j\text{th annulus}$$

and

$$|P_y(t)| \leq Cy^{-n} \text{ on the ball } |t| < \frac{\lambda}{2^N},$$

we obtain exactly in the same way as in [3] that

$$I_1 \leq 2^n C\beta.$$

To estimate I_2 we proceed as follows. Since C_c is dense in $L^1\left(w^{-n-1}\right)$, given $\varepsilon > 0$ we write $f = h + g$ where $g \in C_c$ and $\|h\|_{L^1(w^{-n-1})} < \varepsilon$. Thus

$$\begin{aligned} I_2 &\leq \int_{|t|\geq\lambda} |h(x-t) - h(x)|\, P_y(t)\, dt + \int_{|t|\geq\lambda} |g(x-t) - g(x)|\, P_y(t)\, dt \\ &\leq \int_{|t|\geq\lambda} |h(x-t)|\, P_y(t)\, dt + |h(x)| \int_{|t|\geq\lambda} P_y(t)\, dt + 2\|g\|_\infty \int_{|t|\geq\lambda} P_y(t)\, dt \end{aligned}$$

and the last two terms in the sum go to 0 as $y \to 0^+$.

It remains to analyze the term

$$\int_{|t|\geq\lambda} |h(x-t)|\, P_y(t)\, dt.$$

Using the change of variable $u = y^{-1}t$ we obtain

$$\int_{|t|\geq\lambda} |h(x-t)|\, P_y(t)\, dt = \int_{|u|\geq\lambda/y} |h(x-yu)|\, P(u)\, du$$

$$= C_n \int_{|u|\geq\lambda/y} \frac{|h(x-yu)|}{\left(1+|x-yu|^2\right)^{\frac{n+1}{2}}} \frac{\left(1+|x-yu|^2\right)^{\frac{n+1}{2}}}{\left(1+|u|^2\right)^{\frac{n+1}{2}}}\, du$$

$$\leq C_n \sup_{|u|\geq\lambda/y} \left(\frac{1+|x-yu|^2}{1+|u|^2}\right)^{\frac{n+1}{2}} \int_{\mathbf{R}^n} \frac{|h(x-yu)|}{\left(1+|x-yu|^2\right)^{\frac{n+1}{2}}}\, du$$

$$= C_n \sup_{|u|\geq\lambda/y} \left(\frac{1+|x-yu|^2}{1+|u|^2}\right)^{\frac{n+1}{2}} y^{-n} \int_{\mathbf{R}^n} \frac{|h(s)|}{\left(1+|s|^2\right)^{\frac{n+1}{2}}}\, ds$$

$$\leq C_n \varepsilon y^{-n} \sup_{|u|\geq\lambda/y} \left(\frac{1+|x-yu|^2}{1+|u|^2}\right)^{\frac{n+1}{2}}.$$

We need to check that

$$\sup_{|u|\geq\lambda/y} \left(\frac{1+|x-yu|^2}{1+|u|^2}\right)^{\frac{n+1}{2}} < \infty$$

and

$$C_n \varepsilon y^{-n} \sup_{|u|\geq\lambda/y} \left(\frac{1+|x-yu|^2}{1+|u|^2}\right)^{\frac{n+1}{2}} \to 0$$

as $y \to 0^+$.

We observe that

$$C_n \varepsilon y^{-n} \sup_{|u|\geq\lambda/y} \left(\frac{1+|x-yu|^2}{1+|u|^2}\right)^{\frac{n+1}{2}} = C_n \varepsilon y \sup_{|u|\geq\lambda/y} \left(\frac{1+|x-yu|^2}{y^2+|yu|^2}\right)^{\frac{n+1}{2}}$$

$$\leq C_n \varepsilon y \sup_{|u|\geq\lambda/y} \left(\frac{1+(|x|+|yu|)^2}{y^2+|yu|^2}\right)^{\frac{n+1}{2}}$$

$$\leq C_n \varepsilon y \left(1+|x|^2\right)^{\frac{n+1}{2}} \sup_{|u|\geq\lambda/y} \left(\frac{\left(1+|yu|^2\right)}{y^2+|yu|^2}\right)^{\frac{n+1}{2}}$$

$$= C_n \varepsilon y \left(1+|x|^2\right)^{\frac{n+1}{2}} \sup_{|u|\geq \lambda/y} \left(\frac{\left(\frac{1}{y^2}+|u|^2\right)}{1+|u|^2}\right)^{\frac{n+1}{2}}$$

$$\leq C_n \varepsilon y \left(1+|x|^2\right)^{\frac{n+1}{2}} \sup_{|u|\geq \lambda/y} \left(\frac{\frac{|u|^2}{\lambda^2}+|u|^2}{1+|u|^2}\right)^{\frac{n+1}{2}}$$

$$\leq C_n \varepsilon y \left(\frac{1}{\lambda^2}+1\right)^{\frac{n+1}{2}} \left(1+|x|^2\right)^{\frac{n+1}{2}} \sup_{|u|\geq \lambda/y} \left(\frac{|u|^2}{1+|u|^2}\right)^{\frac{n+1}{2}}$$

$$\leq C_n \varepsilon y \left(\frac{1}{\lambda^2}+1\right)^{\frac{n+1}{2}} \left(1+|x|^2\right)^{\frac{n+1}{2}} \to 0 \text{ as } y \to 0^+.$$

This concludes the proof. ∎

Remark 11. Using the fact that the function $\phi(x) = \frac{1}{\left(1+|x|^2\right)^{s/2}} = w^{-s}$ with $0 < s < n$ is an A_1 weight (see [2, Lemma 5.2], we can assure the boundedness of the Hardy-Littlewood maximal operator M from $L^p(\phi)$ into itself , $1 < p < \infty$, as well as from $L^1(\phi)$ into weak $L^1(\phi)$. This fact, plus the standard estimate $|(P_y * f)(x)| \leq CMf(x)$, and the arguments given in [4, Th. 4.12, p. 177], allow us to prove that $(P_y * f)(x) \to f(x)$ as $y \to 0^+$ for almost every $x \in \mathbf{R}^n$ and $f \in L^p(w^{-s})$, $1 \leq p < \infty$. Since $L^p(w^{-s}) \subset L^p\left(w^{-n-1}\right) \subset L^1\left(w^{-n-1}\right)$ and the inclusions are strict, Theorem 10 enlarge the class of functions for which the pointwise convergence occurs.

Remark 12. Given $T \in w^{n+1}\mathcal{D}'_{L^1}$ we know that $T * P_y \in L^1\left(w^{-n-1}\right)$ for each $y > 0$, thus, it seems natural to ask if $T * P_y$ has pointwise boundary values. However, this is not a relevant question. For example, if we consider the Dirac measure μ concentrated at 0, we have that $P_y = \mu * P_y$ and $P_y \to 0$ as $y \to 0^+$ a.e., thus the boundary value 0 does not determine P_y. In this case $T = \mu \in \mathcal{D}'_{L^1}$.

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