

## COMMON FIXED POINT OF SELF-MAPS IN INTUITIONISTIC FUZZY METRIC SPACES

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**Abstract.** Intuitionistic fuzzy metric spaces have been defined by J.H. Park ([5]). Although topological structure of an intuitionistic fuzzy metric space  $(X, M, N, *, \diamond)$  coincides with the topological structure of the fuzzy metric space  $(X, M, *)$  ([2]), study of common fixed theory in intuitionistic fuzzy metric (and normed) spaces is interesting. We shall give some results in this field.

### 1. Introduction

Motivated by the potential applicability of fuzzy topology to quantum particle physics, Park introduced and discussed in [5] a notion of intuitionistic fuzzy metric space. Actually, Park's notion is useful in modeling some phenomena where it is necessary to study the relationship between two probability functions. Many authors have proved different fixed point theorems in fuzzy metric spaces (for example; [3], [11], [13]) and intuitionistic fuzzy metric spaces (for example; [1], [4], [6], [8]–[10]). In this paper, we shall prove some results about common fixed point theorems in intuitionistic fuzzy metric (and normed) spaces.

DEFINITION 1.1. ([12]) A binary operation  $*$ :  $[0, 1] \times [0, 1] \longrightarrow [0, 1]$  is continuous  $t$ -norm if  $*$  satisfies the following conditions:

- (a)  $*$  is commutative and associative,
- (b)  $*$  is continuous,
- (c)  $a * 1 = a$  for all  $a \in [0, 1]$ ,
- (d)  $a * b \leq c * d$  whenever  $a \leq c$  and  $b \leq d$ , and  $a, b, c, d \in [0, 1]$ .

EXAMPLE 1.2. Two examples of continuous  $t$ -norm are  $a * b = ab$  and  $a * b = \min(a, b)$ .

DEFINITION 1.3. ([12]) A binary operation  $\diamond$ :  $[0, 1] \times [0, 1] \longrightarrow [0, 1]$  is continuous  $t$ -conorm if  $\diamond$  satisfies the following conditions:

- (a)  $\diamond$  is commutative and associative,

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- (b)  $\diamond$  is continuous,
- (c)  $a \diamond 0 = a$  for all  $a \in [0, 1]$ ,
- (d)  $a \diamond b \leq c \diamond d$  whenever  $a \leq c$  and  $b \leq d$ , and  $a, b, c, d \in [0, 1]$ .

EXAMPLE 1.4. Two examples of continuous  $t$ -conorm are  $a \diamond b = \min(a + b, 1)$  and  $a \diamond b = \max(a, b)$ .

DEFINITION 1.5. ([1]) A 5-tuple  $(X, M, N, *, \diamond)$  is said to be an intuitionistic fuzzy metric space if  $X$  is an arbitrary set,  $*$  is a continuous  $t$ -norm,  $\diamond$  is a continuous  $t$ -conorm and  $M, N$  are fuzzy sets on  $X^2 \times [0, \infty)$  satisfying the following conditions;

- (i)  $M(x, y, t) + N(x, y, t) \leq 1$ ,
- (ii)  $M(x, y, 0) = 0$ ,
- (iii)  $M(x, y, t) = 1$  for all  $t > 0$  if and only if  $x = y$ ,
- (iv)  $M(x, y, t) = M(y, x, t)$ ,
- (v)  $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$  for all  $x, y, z \in X, s, t > 0$ ,
- (vi)  $M(x, y, \cdot): [0, \infty) \rightarrow [0, 1]$  is left continuous,
- (vii)  $\lim_{t \rightarrow \infty} M(x, y, t) = 1$  for all  $x, y \in X$ ,
- (viii)  $N(x, y, 0) = 1$ ,
- (ix)  $N(x, y, t) = 0$  for all  $t > 0$  if and only if  $x = y$ ,
- (x)  $N(x, y, t) = N(y, x, t)$ ,
- (xi)  $N(x, y, t) \diamond N(y, z, s) \geq N(x, z, t + s)$  for all  $x, y, z \in X, s, t > 0$ ,
- (xii)  $N(x, y, \cdot): [0, \infty) \rightarrow [0, 1]$  is left continuous,
- (xiii)  $\lim_{t \rightarrow \infty} N(x, y, t) = 0$  for all  $x, y \in X$ .

Then  $(M, N)$  is called an intuitionistic fuzzy metric on  $X$ . The functions  $M(x, y, t)$  and  $N(x, y, t)$  denoted the degree of nearness and the degree of non-nearness between  $x$  and  $y$  with respect to  $t$ , respectively.

Every fuzzy metric space  $(X, M, *)$  is an intuitionistic fuzzy metric space of the form  $(X, M, 1 - M, *, \diamond)$ , where  $x \diamond y = 1 - [(1 - x) * (1 - y)]$  for all  $x, y \in X$ . Also, note that in every intuitionistic fuzzy metric space  $M(x, y, \cdot)$  is non-decreasing and  $N(x, y, \cdot)$  is non-increasing for all  $x, y \in X$ .

Let  $(X, d)$  be a metric space. Denote  $a * b = ab$  and  $a \diamond b = \min(a + b, 1)$  for all  $a, b \in [0, 1]$  and let  $M_d$  and  $N_d$  be fuzzy sets on  $X^2 \times [0, \infty)$  defined as following;

$$M_d(x, y, t) = \frac{ht^n}{ht^n + md(x, y)}, N_d(x, y, t) = \frac{d(x, y)}{kt^n + md(x, y)}$$

for all  $h, k, m, n > 0$ . Then,  $(X, M_d, N_d, *, \diamond)$  is an intuitionistic fuzzy metric space.

In intuitionistic fuzzy metric space  $(X, M, N, *, \diamond)$ , define

$$B(x, r, t) = \{y \in X : M(x, y, t) > 1 - r, N(x, y, t) < r\},$$

for all  $t > 0$  and  $0 < r < 1$ . Let  $\tau_{(M, N)}$  be the set of all  $A \subseteq X$  with  $x \in A$  if and only if there exist  $t > 0$  and  $0 < r < 1$  such that  $B(x, r, t) \subseteq A$ . Then,  $\tau_{(M, N)}$  is a

topology on  $X$  (induced by the intuitionistic fuzzy metric  $(M, N)$ ). This topology is Hausdorff and first countable. A sequence  $\{x_n\}_{n \geq 1}$  in  $X$  converges to  $x$  if and only if  $\lim_{n \rightarrow \infty} M(x_n, x, t) = 1$  and  $\lim_{n \rightarrow \infty} N(x_n, x, t) = 0$  for all  $t > 0$ , and we denote it by  $x_n \rightarrow y$ . It is called a Cauchy sequence if for each  $0 < \varepsilon < 1$  and  $t > 0$ , there exists a natural number  $n_0$  such that  $M(x_n, x_m, t) > 1 - \varepsilon$  and  $N(x_n, x_m, t) < \varepsilon$  for all  $m, n \geq n_0$ . The intuitionistic fuzzy metric space  $(X, M, N, *, \diamond)$  is said to be complete if every Cauchy sequence is convergent. Also, if  $f: X \rightarrow X$  is a self-map, then we say that  $f$  is sequentially continuous whenever  $x_n \rightarrow y$  implies  $f(x_n) \rightarrow f(y)$ , for each convergent sequence  $\{x_n\}_{n \geq 1}$ . In fact, as in metric spaces, continuity of  $f$  is equivalent to sequentially continuity of  $f$ . Here, we recall the definition of fuzzy metric spaces.

DEFINITION 1.6. A 3-tuple  $(X, M, N, *, \diamond)$  is said to be a fuzzy metric space if  $X$  is an arbitrary set,  $*$  is a continuous  $t$ -norm and  $M$  is a fuzzy set on  $X^2 \times [0, \infty)$  satisfying the following conditions;

- (i)  $M(x, y, t) > 0$ ,
- (ii)  $M(x, y, t) = 1$  for all  $t > 0$  if and only if  $x = y$ ,
- (iii)  $M(x, y, t) = M(y, x, t)$ ,
- (iv)  $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$  for all  $x, y, z \in X, s, t > 0$ ,
- (v)  $M(x, y, \cdot): (0, \infty) \rightarrow [0, 1]$  is continuous.

We define the neighborhoods  $B(x, r, t) = \{y \in X : M(x, y, t) > 1 - r\}$ , for all  $t > 0$  and  $0 < r < 1$ . The family of these neighborhoods generates a Hausdorff topology on  $X$  which we denote it by  $\tau_M$ . In 2006, Gregori, Romaguera and Veeramani proved that the topologies  $\tau_{(M, N)}$  and  $\tau_M$  coincide on  $X$  ([2; Theorem 2]). In side of the fact, study of common fixed theory in intuitionistic fuzzy metric spaces is interesting. In the sequel, We will need to the definition of intuitionistic fuzzy normed spaces.

DEFINITION 1.7. ([7]) The 5-tuple  $(X, \mu, \nu, *, \diamond)$  is said to be an intuitionistic fuzzy normed space if  $X$  is a vector space,  $*$  is a continuous  $t$ -norm,  $\diamond$  is a continuous  $t$ -conorm and  $\mu, \nu$  are fuzzy sets on  $X \times (0, \infty)$  satisfying the following conditions for every  $x, y \in X$  and  $t, s > 0$ ;

- (i)  $\mu(x, t) + \nu(x, t) \leq 1$ ,
- (ii)  $\mu(x, t) > 0$ ,
- (iii)  $\mu(x, t) = 1$  if and only if  $x = 0$ ,
- (iv)  $\mu(\alpha x, t) = \mu(x, \frac{t}{|\alpha|})$  for each  $\alpha \neq 0$ ,
- (v)  $\mu(x, t) * \mu(y, s) \leq \mu(x + y, t + s)$ ,
- (vi)  $\mu(x, \cdot): (0, \infty) \rightarrow [0, 1]$  is continuous,
- (vii)  $\lim_{t \rightarrow \infty} \mu(x, t) = 1$  and  $\lim_{t \rightarrow 0} \mu(x, t) = 0$ ,
- (viii)  $\nu(x, t) < 1$ ,
- (ix)  $\nu(x, t) = 0$  if and only if  $x = 0$ ,
- (x)  $\nu(\alpha x, t) = \nu(x, \frac{t}{|\alpha|})$  for each  $\alpha \neq 0$ ,

- (xi)  $\nu(x, t) \diamond \nu(y, s) \geq \nu(x + y, t + s)$ ,
- (xii)  $\nu(x, \cdot) : (0, \infty) \rightarrow [0, 1]$  is continuous,
- (xiii)  $\lim_{t \rightarrow \infty} \nu(x, t) = 0$  and  $\lim_{t \rightarrow 0} \nu(x, t) = 1$ .

Then  $(\mu, \nu)$  is called an intuitionistic fuzzy norm on  $X$ .

Note that every intuitionistic fuzzy normed space is an intuitionistic fuzzy metric space. We say that a subset  $E$  of  $X$  is star-shaped respect to  $p \in X$  whenever

$$\gamma t + (1 - \gamma)p \in E$$

for all  $t \in E$  and  $\gamma \in [0, 1]$ .

## 2. Main Results

Let  $(X, M, N, *, \diamond)$  be an intuitionistic fuzzy metric space. We shall denote the set of fixed points of a self-map  $f: X \rightarrow X$  by  $Fix(f)$ , that is

$$Fix(f) = \{x \in X : f(x) = x\}.$$

**THEOREM 2.1.** *Let  $(X, M, N, *, \diamond)$  be an intuitionistic fuzzy metric space,  $K > 0$  and  $f, g$  two self-maps with the property  $M(g(f(x)), f(g(x)), Kt) \geq M(g(x), x, t)$  for all  $x \in X$  and  $t > 0$ , or  $N(g(f(x)), f(g(x)), Kt) \leq N(g(x), x, t)$  for all  $x \in X$  and  $t > 0$ . Then,  $f(g(y)) = g(f(y))$  for all  $y \in Fix(g)$ .*

*Proof.* Fix  $y \in Fix(g)$  and let  $M(g(f(x)), f(g(x)), Kt) \geq M(g(x), x, t)$  for all  $x \in X$  and  $t > 0$ . Since  $M(g(y), y, t) = 1$ ,  $M(g(f(y)), f(g(y)), Kt) = 1$  for all  $t > 0$ . Hence,  $g(f(y)) = f(g(y))$  for all  $y \in Fix(g)$ . ■

Let  $(X, M, N, *, \diamond)$  be an intuitionistic fuzzy metric space,  $f, g$  two self-maps and  $A = \{x \in X : f(x) = g(x)\}$ . It is easy to see that  $f(g(y)) = g(f(y))$  for all  $y \in A$  if and only if  $f(y) \in A$  for all  $y \in A$ .

**PROPOSITION 2.2.** *Let  $(X, M, N, *, \diamond)$  be an intuitionistic fuzzy metric space and  $f, g$  two sequentially continuous self-maps on  $X$ . Then,  $f(Fix(g)) \subseteq Fix(g)$  if and only if for each sequence  $\{x_n\}_{n \geq 1}$  in  $X$  such that*

$$\lim_{n \rightarrow \infty} M(g(x_n), y, t) = \lim_{n \rightarrow \infty} M(x_n, y, t) = 1$$

for some  $y \in X$  and for all  $t > 0$ , we have

$$\lim_{n \rightarrow \infty} M(g(f(x_n)), f(x_n), t) = \lim_{n \rightarrow \infty} M(g(f(x_n)), f(g(x_n)), t) = 1.$$

*Proof.* First suppose that  $f(Fix(g)) \subseteq Fix(g)$  and  $\{x_n\}_{n \geq 1}$  is a sequence in  $X$  such that

$$\lim_{n \rightarrow \infty} M(g(x_n), y, t) = \lim_{n \rightarrow \infty} M(x_n, y, t) = 1$$

and

$$\lim_{n \rightarrow \infty} N(g(x_n), y, t) = \lim_{n \rightarrow \infty} N(x_n, y, t) = 0,$$

for some  $y \in X$  and for all  $t > 0$ . Since  $g$  is sequentially continuous,  $y \in \text{Fix}(g)$ . Thus, from sequentially continuity of  $f$  and  $g$  we have

$$g(f(x_n)) \longrightarrow g(f(y)) = f(y), \quad f(g(x_n)) \longrightarrow f(y) \quad \text{and} \quad f(x_n) \longrightarrow f(y).$$

Conversely, let  $x \in \text{Fix}(g)$  and  $x_n = x$  for all  $n \geq 1$ . Then,

$$\lim_{n \rightarrow \infty} M(g(x_n), x, t) = \lim_{n \rightarrow \infty} M(x_n, x, t) = 1$$

and

$$\lim_{n \rightarrow \infty} N(g(x_n), x, t) = \lim_{n \rightarrow \infty} N(x_n, x, t) = 0,$$

for all  $t > 0$ . Hence by assumption,

$$g(f(x)) = f(x) \quad \text{and} \quad g(f(x)) = f(g(x)).$$

Thus,  $f(\text{Fix}(g)) \subseteq \text{Fix}(g)$ . ■

LEMMA 2.3. *Let  $(X, M, N, *, \diamond)$  be a complete intuitionistic fuzzy metric space,  $K$  a number in  $(0, 1)$  and  $f, g$  two self-maps on  $X$  such that  $\text{Fix}(g)$  is non-empty and closed,  $f(\text{Fix}(g)) \subseteq \text{Fix}(g)$  and*

$$M(f(x), f(y), Kt) \geq M(g(x), g(y), t)$$

for all  $x, y \in X$  and  $t > 0$ . Then, the set of common fixed points of  $f$  and  $g$ ,  $\text{Fix}(f, g) := \{x \in X : f(x) = g(x) = x\}$ , is singleton.

*Proof.* Since  $\text{Fix}(g)$  is a non-empty closed subset of  $X$ ,  $\text{Fix}(g)$  is complete. Hence, the restriction of  $f$  on  $\text{Fix}(g)$  satisfies the relations

$$M(f(x), f(y), Kt) \geq M(g(x), g(y), t) = M(x, y, t)$$

for all  $x, y \in \text{Fix}(g)$  and  $t > 0$ . Thus by [1; Theorem 7], the restriction of  $f$  on  $\text{Fix}(g)$  has an unique fixed point  $x_0$ . Therefore,  $\text{Fix}(f, g) = \{x_0\}$ . ■

THEOREM 2.4. *Let  $(X, \mu, \nu, *, \diamond)$  be an intuitionistic fuzzy normed space,  $f, g$  two sequentially continuous self-maps such that  $p \in \text{Fix}(g)$ ,  $\text{Fix}(g)$  closed and star-shaped respect to  $p$ , closure of  $f(X)$  compact,  $f(\text{Fix}(g)) \subseteq \text{Fix}(g)$  and  $\mu(f(x) - f(y), t) \geq \mu(g(x) - g(y), t)$  for all  $x \in X$  and  $t > 0$ . Then,  $\text{Fix}(f, g)$  is non-empty.*

*Proof.* Let  $\{\lambda_n\}_{n \geq 1}$  be a sequence in  $(0, 1)$  such that  $\lim_{n \rightarrow \infty} \lambda_n = 1$ . For each  $n \geq 1$ , define  $f_n : X \longrightarrow X$  by  $f_n(x) = \lambda_n f(x) + (1 - \lambda_n)p$ . Since  $f(\text{Fix}(g)) \subseteq \text{Fix}(g)$  and  $\text{Fix}(g)$  is star-shaped respect to  $p$ ,  $f_n(\text{Fix}(g)) \subseteq \text{Fix}(g)$  for all  $n \geq 1$ . Also,  $\mu(f_n(x) - f_n(y), \lambda_n t) = \mu(\lambda_n(f(x) - f(y)), \lambda_n t) \geq \mu(g(x) - g(y), t)$  for all  $x, y \in X$ ,  $n \geq 1$  and  $t > 0$ . Hence by Lemma 2.3,  $\text{Fix}(f_n, g) = \{x_n\}$  for some  $x_n \in X$ . But by compactness of closure of  $f(X)$ , there exists a subsequence  $\{x_{n_i}\}_{i \geq 1}$  such that  $\{f(x_{n_i})\}_{i \geq 1}$  converges to some element  $x_0$  in closure of  $f(X)$ . Since

$$x_{n_i} = f_{n_i}(x_{n_i}) = \lambda_{n_i} f(x_{n_i}) + (1 - \lambda_{n_i})p \rightarrow x_0$$

and  $g(x_{n_i}) = x_{n_i}$ , the continuity of  $f$  and  $g$  implies  $g(x_0) = x_0$  and  $f(x_0) = x_0$ . Therefore,  $x_0 \in \text{Fix}(f, g)$ . ■

The following examples show that there are some applications for common fixed point theory.

EXAMPLE 2.1. Let  $X = [0, \infty)$ ,  $d(x, y) = |x - y|$ ,  $a * b = \min\{a, b\}$ ,  $a \diamond b = \max\{a, b\}$  (for all  $a, b \in [0, 1]$ ),  $M(x, y, t) = \frac{t}{t+d(x,y)}$ ,  $N(x, y, t) = \frac{d(x,y)}{t+d(x,y)}$  and the self-maps  $f, g : X \rightarrow X$  defined by

$$f(x) = 2x, \quad g(x) = \frac{x}{1+x}.$$

Note that,  $f(\text{Fix}(g)) \subseteq \text{Fix}(g)$  and there is not any  $K > 0$  such that

$$M(f(x), f(y), Kt) \geq M(g(x), g(y), t)$$

holds for all  $x, y \in X$  and  $t > 0$ . But,  $\text{Fix}(f, g) = \{0\}$ .

EXAMPLE 2.2. Let  $X = \mathbf{R}$ ,  $d(x, y) = |x - y|$ ,  $a * b = \min\{a, b\}$ ,  $a \diamond b = \max\{a, b\}$  (for all  $a, b \in [0, 1]$ ),  $M(x, y, t) = \frac{t}{t+d(x,y)}$ ,  $N(x, y, t) = \frac{d(x,y)}{t+d(x,y)}$  and  $\varphi_1, \varphi_2 : X \rightarrow X$  two self-maps. We know that the systems  $\begin{cases} \varphi_1(x) = 0 \\ \varphi_2(x) = 0 \end{cases}$  and  $\begin{cases} \psi_1(x) = x \\ \psi_2(x) = x \end{cases}$  have the same solutions, where  $\psi_1(x) = \varphi_1(x) + x$  and  $\psi_2(x) = \varphi_2(x) + x$ . Thus if the self maps  $\psi_1$  and  $\psi_2$  satisfy the relations  $\psi_1(\text{Fix}(\psi_2)) \subseteq \text{Fix}(\psi_2)$  and

$$M(\psi_1(x), \psi_1(y), Kt) \geq M(\psi_2(x), \psi_2(y), t)$$

for some  $K \in (0, 1)$  and all  $x, y \in X$ ,  $t > 0$ , then by Lemma 2.3, the system  $\begin{cases} \varphi_1(x) = 0 \\ \varphi_2(x) = 0 \end{cases}$  has a unique solution.

EXAMPLE 2.3. Let  $X = \mathbf{R}^n$ ,  $a * b = \min\{a, b\}$ ,  $a \diamond b = \max\{a, b\}$  (for all  $a, b \in [0, 1]$ ),  $\mu(\mathbf{x}, \mathbf{y}, t) = \frac{t}{t+\|\mathbf{x}-\mathbf{y}\|}$ ,  $\nu(\mathbf{x}, \mathbf{y}, t) = \frac{\|\mathbf{x}-\mathbf{y}\|}{t+\|\mathbf{x}-\mathbf{y}\|}$  and  $\varphi_1, \varphi_2 : X \rightarrow X$  two self-maps. Put  $\psi_1(\mathbf{x}) = \varphi_1(\mathbf{x}) + \mathbf{x}$  and  $\psi_2(\mathbf{x}) = \varphi_2(\mathbf{x}) + \mathbf{x}$ . If the self maps  $\psi_1$  and  $\psi_2$  are sequentially continuous,  $p \in \text{Fix}(\psi_2)$ ,  $\text{Fix}(\psi_2)$  is closed and star-shaped respect to  $p$ , closure of  $\psi_1(X)$  compact,  $\psi_1(\text{Fix}(\psi_2)) \subseteq \text{Fix}(\psi_2)$  and

$$\mu(\psi_1(\mathbf{x}) - \psi_1(\mathbf{y}), t) \geq \mu(\psi_2(\mathbf{x}) - \psi_2(\mathbf{y}), t)$$

for all  $\mathbf{x}, \mathbf{y} \in X$  and  $t > 0$ , then by Theorem 2.4, the system  $\begin{cases} \varphi_1(\mathbf{x}) = 0 \\ \varphi_2(\mathbf{x}) = 0 \end{cases}$  has at least one solution.

EXAMPLE 2.4. Let  $X = C_{\mathbf{R}}([0, 1])$ ,  $d(f, g) = \sup_{x \in [0, 1]} |f(x) - g(x)|$ ,  $a * b = \min\{a, b\}$ ,  $a \diamond b = \max\{a, b\}$  (for all  $a, b \in [0, 1]$ ),  $M(f, g, t) = \frac{t}{t+d(f,g)}$ ,  $N(f, g, t) = \frac{d(f,g)}{t+d(f,g)}$  and  $\varphi, \psi : X \rightarrow X$  two self-maps defined by

$$\varphi(f)(x) = \int_0^x \frac{1}{2} f(t) dt, \quad \psi(f) = f,$$

for all  $f, g \in X$  and  $x \in [0, 1]$ . Note that,  $\varphi(\text{Fix}(\psi)) \subseteq \text{Fix}(\psi)$  and

$$M(\varphi(f), \varphi(g), \frac{1}{2}t) \geq M(\psi(f), \psi(g), t)$$

for all  $f, g \in X$  and  $t > 0$ . By Lemma 2.3, the equation

$$\varphi(f)(x) = \int_0^x \frac{1}{2}f(t) dt = f(x), \text{ for all } x \in [0, 1],$$

has not solution except  $f = 0$ .

This example show that we can solve some integral equations by using common fixed point theory.

### 3. Conclusions

This paper presents two common fixed point results about common fixed point self-mappings on intuitionistic fuzzy metric spaces (Lemma 2.3) and intuitionistic fuzzy normed spaces (Theorem 2.4) with some contractive conditions. Also, there is a result about conditions of commuting of two self-maps on intuitionistic fuzzy metric spaces (Theorem 2.1) and another one about equivalent condition of a basic condition which has important role in the results of this paper (Proposition 2.2). The example 2.1 shows that there are self-maps which don't satisfies in the contractive condition but those satisfy in the result of Lemma 2.3. The examples 2.2 and 2.3 provide an important application of common fixed point theory in solving of systems of equations in  $\mathbf{R}^n$ . This shows that common fixed point theory could be used for solving of systems of differential equations. Finally, the example 2.4 provides another application of the theory in solving of some integral equations. In general view, the results of this paper guarantee existence of common fixed points, and so in application view, guarantee existence of solution for systems of equations.

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