МАТЕМАТИЧКИ ВЕСНИК
64, 3 (2012), 258–266
September 2012

оригинални научни рад
research paper

A GENERALIZATION OF FIXED POINT THEOREMS IN S-METRIC SPACES

Shaban Sedghi, Nabi Shobe and Abdelkrim Aliouche

Abstract. In this paper, we introduce S-metric spaces and give some of their properties. Also we prove a fixed point theorem for a self-mapping on a complete S-metric space.

1. Introduction

Metric spaces are very important in mathematics and applied sciences. So, some authors have tried to give generalizations of metric spaces in several ways. For example, Gähler [3] and Dhage [2] introduced the concepts of 2-metric spaces and D-metric spaces, respectively, but some authors pointed out that these attempts are not valid (see [6–10]).

Mustafa and Sims [4] introduced a new structure of generalized metric spaces which are called G-metric spaces as a generalization of metric spaces (X, d) to develop and introduce a new fixed point theory for various mappings in this new structure. Some authors [1, 5, 13] have proved some fixed point theorems in these spaces.

Recently, Sedghi et al. [12] have introduced D^*-metric spaces which is a probable modification of the definition of D-metric spaces introduced by Dhage [2] and proved some basic properties in D^*-metric spaces, (see [11, 12]).

In the present paper, we introduce the concept of S-metric spaces and give some of their properties. Then a common fixed point theorem for a self-mapping on complete S-metric spaces is given.

We begin with the following definitions:

Definition 1.1. [4] Let X be a nonempty set and $G : X \times X \times X \to [0, \infty)$ be a function satisfying the following conditions for all $x, y, z, a \in X$,

(G1) $G(x, y, z) = 0$ if $x = y = z$,

(G2) $0 < G(x, x, y)$ for all $x, y \in X$ with $x \neq y$,

2010 AMS Subject Classification: 54H25; 47H10.
Keywords and phrases: S-metric space; fixed point.

(G3) $G(x, x, y) \leq G(x, y, z)$ for all $x, y, z \in X$ with $x \neq y$,

(G4) $G(x, y, z) = G(x, z, y) = G(y, z, x) = \cdots$,

(G5) $G(x, y, z) \leq G(x, a, a) + G(a, y, z)$ for all $x, y, x, a \in X$.

Then the function G is called a *generalized metric* or a G-*metric*on X and the pair (X, G) is called a G-metric space.

We can find some examples and basic properties of G-metric spaces in Mustafa and Sims [4].

Definition 1.2 [12] Let X be a nonempty set. A generalized metric (or D^*-*metric*) on X is a function: $D^* : X^3 \to \mathbb{R}^+$ that satisfies the following conditions for each $x, y, z, a \in X$.

(1) $D^*(x, y, z) \geq 0$,

(2) $D^*(x, y, z) = 0$ if and only if $x = y = z$,

(3) $D^*(x, y, z) = D^*(p\{x, y, z\})$, (symmetry), where p is a permutation function,

(4) $D^*(x, y, z) \leq D^*(x, y, a) + D^*(a, z, z)$.

The pair (X, D^*) is called a generalized metric (or D^*-metric) space.

Immediate examples of such functions are:

(a) $D^*(x, y, z) = \max\{d(x, y), d(y, z), d(z, x)\}$,

(b) $D^*(x, y, z) = d(x, y) + d(y, z) + d(z, x)$.

Here, d is the ordinary metric on X.

(c) If $X = \mathbb{R}^n$ then we define

$$D^*(x, y, z) = ||x + y - 2z|| + ||x + z - 2y|| + ||y + z - 2x||.$$

(d) If $X = \mathbb{R}^+$ then we define

$$D^*(x, y, z) = \begin{cases} 0 & \text{if } x = y = z, \\ \max\{x, y, z\} & \text{otherwise,} \end{cases}$$

Remark 1.3. It is easy to see that every G-metric is a D^*-metric, but in general the converse does not hold, see the following example.

Example 1.4. If $X = \mathbb{R}$, we define

$$D^*(x, y, z) = |x + y - 2z| + |x + z - 2y| + |y + z - 2x|.$$

It is easy to see that $(\mathbb{R}, D^*)$ is a D^*-metric, but it is not G-metric. Set $x = 5$, $y = -5$ and $z = 0$ then $G(x, x, y) \leq G(x, y, z)$ does not hold.

Now, we introduce the concept of S-metric spaces which modifies D-metric and G-metric spaces.

2. S-metric spaces

We begin with the following definition.

Definition 2.1. Let X be a nonempty set. An *S-metric* on X is a function $S : X^3 \to [0, \infty)$ that satisfies the following conditions, for each $x, y, z, a \in X$,

(1) $S(x, y, z) \geq 0$,

(2) $S(x, y, z) = 0$ if and only if $x = y = z$,

(3) $S(x, y, z) \leq S(x, x, a) + S(y, y, a) + S(z, z, a)$

The pair (X, S) is called an *S-metric space*.

Immediate examples of such S-metric spaces are:

(1) Let $X = \mathbb{R}^n$ and $\|\cdot\|$ a norm on X, then $S(x, y, z) = \|y + z - 2x\| + \|y - z\|$ is an S-metric on X.

(2) Let $X = \mathbb{R}^n$ and $\|\cdot\|$ a norm on X, then $S(x, y, z) = \|x - z\| + \|y - z\|$ is an S-metric on X.

(3) Let X be a nonempty set, d is ordinary metric on X, then $S(x, y, z) = d(x, z)+ d(y, z)$ is an S-metric on X.

Remark 2.2. It is easy to see that every D^*-metric is S-metric, but in general the converse is not true, see the following example.

Example 2.3. Let $X = \mathbb{R}^n$ and $\|\cdot\|$ a norm on X, then $S(x, y, z) = \|y + z - 2x\| + \|y - z\|$ is S-metric on X, but it is not D^*-metric because it is not symmetric.

Example 2.4. [intuitive geometric example for S-metric] Let $X = \mathbb{R}^2$, d is an ordinary metric on X, therefore, $S(x, y, z) = d(x, y) + d(x, z) + d(y, z)$ is an S-metric on X. If we connect the points x, y, z by a line, we have a triangle and if we choose a point a mediating this triangle then the inequality $S(x, y, z) \leq S(x, x, a) + S(y, y, a) + S(z, z, a)$ holds. In fact

$$\begin{aligned} S(x, y, z) &= d(x, y) + d(x, z) + d(y, z) \\ &\leq d(x, a) + d(a, y) + d(x, a) + d(a, z) + d(y, a) + d(a, z) \\ &= S(x, x, a) + S(y, y, a) + S(z, z, a). \end{aligned}$$

Lemma 2.5. *In an S-metric space, we have $S(x, x, y) = S(y, y, x)$.*

Proof. By the third condition of S-metric, we get

$$S(x, x, y) \leq S(x, x, x) + S(x, x, x) + S(y, y, x) = S(y, y, x) \quad (1)$$

and similarly

$$S(y, y, x) \leq S(y, y, y) + S(y, y, y) + S(x, x, y) = S(x, x, y). \quad (2)$$

Hence, by (1) and (2), we obtain $S(x, x, y) = S(y, y, x)$. ■

Definition 2.6. Let (X, S) be an S-metric space. For $r > 0$ and $x \in X$ we define the open ball $B_S(x, r)$ and closed ball $B_S[x, r]$ with a center x and a radius r as follows:

$$\begin{aligned} B_S(x, r) &= \{y \in X : S(y, y, x) < r\}, \\ B_S[x, r] &= \{y \in X : S(y, y, x) \leq r\}. \end{aligned}$$

EXAMPLE 2.7. Let $X = \mathbb{R}$. Denote $S(x, y, z) = |y + z - 2x| + |y - z|$ for all $x, y, z \in \mathbb{R}$. Therefore

$$B_S(1, 2) = \{y \in \mathbb{R} : S(y, y, 1) < 2\} = \{y \in \mathbb{R} : |y - 1| < 1\} = (0, 2).$$

DEFINITION 2.8. Let (X, S) be an S-metric space and $A \subset X$.

(1) If for every $x \in A$ there exists $r > 0$ such that $B_S(x, r) \subset A$, then the subset A is called an open subset of X.

(2) A subset A of X is said to be S-*bounded* if there exists $r > 0$ such that $S(x, x, y) < r$ for all $x, y \in A$.

(3) A sequence $\{x_n\}$ in X converges to x if and only if $S(x_n, x_n, x) \to 0$ as $n \to \infty$. That is for each $\varepsilon > 0$ there exists $n_0 \in \mathbb{N}$ such that for all $n \geq n_0$, $S(x_n, x_n, x) < \varepsilon$ and we denote this by $\lim_{n\to\infty} x_n = x$.

(4) A sequence $\{x_n\}$ in X is called a *Cauchy sequence* if for each $\varepsilon > 0$, there exists $n_0 \in \mathbb{N}$ such that $S(x_n, x_n, x_m) < \varepsilon$ for each $n, m \geq n_0$.

(5) The S-metric space (X, S) is said to be *complete* if every Cauchy sequence is convergent.

(6) Let τ be the set of all $A \subset X$ with $x \in A$ if and only if there exists $r > 0$ such that $B_S(x, r) \subset A$. Then τ is a topology on X (induced by the S-metric S).

LEMMA 2.9. *Let (X, S) be an S-metric space. If $r > 0$ and $x \in X$, then the ball $B_S(x, r)$ is an open subset of X.*

Proof. Let $y \in B_S(x, r)$, hence $S(y, y, x) < r$. If we set $\delta = S(x, x, y)$ and $r' = \frac{r-\delta}{2}$ then we prove that $B_S(y, r') \subseteq B_S(x, r)$. Let $z \in B_S(y, r')$, therefore, $S(z, z, y) < r'$. By the third condition of S-metric we have

$$S(z, z, x) \leq S(z, z, y) + S(z, z, y) + S(x, x, y) < 2r' + \delta = r$$

and so $B_S(y, r') \subseteq B_S(x, r)$. ■

LEMMA 2.10. *Let (X, S) be an S-metric space. If the sequence $\{x_n\}$ in X converges to x, then x is unique.*

Proof. Let $\{x_n\}$ converges to x and y. Then for each $\varepsilon > 0$ there exist $n_1, n_2 \in \mathbb{N}$ such that

$$n \geq n_1 \Longrightarrow S(x_n, x_n, x) < \frac{\varepsilon}{2}$$

and

$$n \geq n_2 \Longrightarrow S(x_n, x_n, y) < \frac{\varepsilon}{2}.$$

If set $n_0 = \max\{n_1, n_2\}$, therefore for every $n \geq n_0$ and the third condition of S-metric we get

$$S(x, x, y) \leq 2S(x, x, x_n) + S(y, y, x_n) < \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon.$$

Hence $S(x, x, y) = 0$ and so $x = y$. ■

Lemma 2.11. *Let (X, S) be an S-metric space. If the sequence $\{x_n\}$ in X converges to x, then $\{x_n\}$ is a Cauchy sequence.*

Proof. Since $\lim_{n\to\infty} x_n = x$ then for each $\varepsilon > 0$ there exists $n_1, n_2 \in \mathbb{N}$ such that

$$n \geq n_1 \Rightarrow S(x_n, x_n, x) < \frac{\varepsilon}{4}$$

and

$$m \geq n_2 \Rightarrow S(x_m, x_m, x) < \frac{\varepsilon}{2}.$$

If we set $n_0 = \max\{n_1, n_2\}$, therefore for every $n, m \geq n_0$ we get by the third condition of S-metric

$$S(x_n, x_n, x_m) \leq 2S(x_n, x_n, x) + S(x_m, x_m, x) < \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon.$$

Hence, $\{x_n\}$ is a Cauchy sequence. ■

Lemma 2.12. *Let (X, S) be an S-metric space. If there exist sequences $\{x_n\}$ and $\{y_n\}$ such that $\lim_{n\to\infty} x_n = x$ and $\lim_{n\to\infty} y_n = y$, then $\lim\limits_{n\to\infty} S(x_n, x_n, y_n) = S(x, x, y)$.*

Proof. Since $\lim_{n\to\infty} x_n = x$ and $\lim_{n\to\infty} y_n = y$, then for each $\varepsilon > 0$ there exist $n_1, n_2 \in \mathbb{N}$ such that

$$\forall\, n \geq n_1, \quad S(x_n, x_n, x) < \frac{\varepsilon}{4}$$

and

$$\forall\, n \geq n_2, \quad S(y_n, y_n, y) < \frac{\varepsilon}{4}.$$

If set $n_0 = \max\{n_1, n_2\}$, therefore for every $n \geq n_0$ we get by the third condition of S-metric

$$\begin{aligned} S(x_n, x_n, y_n) &\leq 2S(x_n, x_n, x) + S(y_n, y_n, x) \\ &\leq 2S(x_n, x_n, x) + 2S(y_n, y_n, y) + S(x, x, y) \\ &< \frac{\varepsilon}{2} + \frac{\varepsilon}{2} + S(x, x, y) = \varepsilon + S(x, x, y). \end{aligned}$$

Hence we obtain

$$S(x_n, x_n, y_n) - S(x, x, y) < \varepsilon. \tag{3}$$

On the other hand, we get

$$\begin{aligned} S(x, x, y) &\leq 2S(x, x, x_n) + S(y, y, x_n) \\ &\leq 2S(x, x, x_n) + 2S(y, y, y_n) + S(x_n, x_n, y_n) \\ &< \frac{\varepsilon}{2} + \frac{\varepsilon}{2} + S(x_n, x_n, y_n) = \varepsilon + S(x_n, x_n, y_n), \end{aligned}$$

that is

$$S(x, x, y) - S(x_n, x_n, y_n) < \varepsilon. \tag{4}$$

Therefore by relations (3) and (4) we have $|S(x_n, x_n, y_n) - S(x, x, y)| < \varepsilon$, that is

$$\lim_{n\to\infty} S(x_n, x_n, y_n) = S(x, x, y). \quad ■$$

Definition 2.13. Let (X, S) be an S-metric space. A map $F : X \to X$ is said to be a contraction if there exists a constant $0 \leq L < 1$ such that

$$S(F(x), F(x), F(y)) \leq L\ S(x, x, y), \text{ for all } \ x, y \in X.$$

3. A generalization of fixed point theorems in S-metric spaces

Note that a contraction map is necessarily continuous because if $x_n \to x$ in the above condition we get $F(x_n) \to F(x)$.

For notational purposes we define $F^n(x), x \in X$ and $n \in \{0, 1, 2, \dots\}$, inductively by $F^0(x) = x$ and $F^{n+1}(x) = F(F^n(x))$.

The first result in this section is known as a similar Banach's contraction principle.

THEOREM 3.1. *Let (X, S) be a complete S-metric space and $F : X \to X$ be a contraction. Then F has a unique fixed point $u \in X$. Furthermore, for any $x \in X$ we have* $\lim_{n\to\infty} F^n(x) = u$ *with*

$$S(F^n(x), F^n(x), u) \le \frac{2L^n}{1-L} S(x, x, F(x)).$$

Proof. First, we show the uniqueness. Suppose that there exist $x, y \in X$ with $x = F(x)$ and $y = F(y)$. Then

$$S(x, x, y) = S(F(x), F(x), F(y)) \le L\ S(x, x, y)$$

and therefore $S(x, x, y) = 0$.

To show the existence, we select $x \in X$ and show that $\{F^n(x)\}$ is a Cauchy sequence. For $n = 0, 1, \dots$, we get by induction

$$\begin{aligned} S(F^n(x), F^n(x), F^{n+1}(x)) &\le L\ S(F^{n-1}(x), F^{n-1}(x), F^n(x)) \\ &\ \vdots \\ &\le L^n\ S(x, x, F(x)). \end{aligned}$$

Thus for $m > n$ we have

$$\begin{aligned} S(F^n(x),&F^n(x), F^m(x)) \\ &\le 2\sum_{i=n}^{m-2} S(F^i(x), F^i(x), F^{i+1}(x)) + S(F^{m-1}(x), F^{m-1}(x), F^m(x)) \\ &\le 2\sum_{i=n}^{m-2} L^i\ S(x, x, F(x)) + L^{m-1}\ S(x, x, F(x)) \\ &\le 2L^n\ S(x, x, F(x))[1 + L + L^2 + \cdots] \\ &\le \frac{2L^n}{1-L}\ S(x, x, F(x)). \end{aligned}$$

That is for $m > n$,

$$S(F^n(x), F^n(x), F^m(x)) \le \frac{2L^n}{1-L}\ S(x, x, F(x)). \tag{5}$$

This shows that $\{F^n(x)\}$ is a Cauchy sequence and since X is complete there exists $u \in X$ with $\lim_{n\to\infty} F^n(x) = u$. Moreover, the continuity of F yields

$$u = \lim_{n\to\infty} F^{n+1}(x) = \lim_{n\to\infty} F(F^n(x)) = Fu.$$

Therefore, u is a fixed point of F. Finally letting $m \to \infty$ in (5) we obtain

$$S(F^n(x), F^n(x), u) \leq \frac{2L^n}{1-L} S(x, x, F(x)). \quad \blacksquare$$

EXAMPLE 3.2. Let $X = R$, then $S(x, y, z) = |x - z| + |y - z|$ is an S-metric on X. Define a self-map F on X by: $F(x) = \frac{1}{2}\sin x$. We have

$$\begin{aligned} S(Fx, Fx, Fy) &= |\frac{1}{2}(\sin x - \sin y)| + |\frac{1}{2}(\sin x - \sin y)| \\ &\leq \frac{1}{2}(|x - y| + |x - y|) = \frac{1}{2} S(x, x, y) \end{aligned}$$

for every $x, y \in X$. Furthermore, for any $x \in X$ we have $\lim_{n\to\infty} F^n(x) = 0$ with

$$S(F^n(x), F^n(x), 0) \leq \frac{2L^n}{1-L} S(x, x, F(x)),\ L = \frac{1}{2}.$$

It follows that all conditions of Theorem 3.1 hold and there exists $u = 0 \in X$ such that $u = Fu$.

THEOREM 3.3. *Let (X, S) be a compact S-metric space with $F : X \to X$ satisfying*

$$S(F(x), F(x), F(y)) < S(x, x, y) \quad \textit{for all} \quad x, y \in X \quad \textit{and} \quad x \neq y.$$

Then F has a unique fixed point in X.

Proof. The uniqueness part is easy. To show the existence, notice that the map $x \mapsto S(x, x, F(x))$ attains its minimum, say at $x_0 \in X$. We have $x_0 = F(x_0)$ since otherwise

$$S(F(F(x_0)), F(F(x_0)), F(x_0)) < S(F(x_0), F(x_0), x_0) = S(x_0, x_0, F(x_0))$$

which is a contradiction. ■

Next, we present a local version of Banach's contraction principle.

THEOREM 3.4. *Let (X, S) be a complete S-metric space and let*

$$B_S(x_0, r) = \{x \in X : S(x, x, x_0) < r\}, \ \textit{where} \ \ x_0 \in X \ \ \textit{and} \ \ r > 0.$$

Suppose that $F : B_S(x_0, r) \to X$ is a contraction with

$$S(F(x_0), F(x_0), x_0) < (1 - L)\frac{r}{2}.$$

Then F has a unique fixed point in $B_S(x_0, r)$.

Proof. There exists r_0 with $0 \leq r_0 < r$ such that $S(F(x_0), F(x_0), x_0) \leq (1 - L)\frac{r_0}{2}$. We will show that $F : \overline{B_S(x_0, r_0)} \to \overline{B_S(x_0, r_0)}$. To see this, note that if $x \in \overline{B_S(x_0, r_0)}$, then

$$\begin{aligned} S(x_0, x_0, F(x)) &\leq 2S(x_0, x_0, F(x_0)) + S(F(x_0), F(x_0), F(x)) \\ &\leq 2(1 - L)\frac{r_0}{2} + L\ S(x_0, x_0, x) \leq r_0. \end{aligned}$$

We can now apply Theorem 3.1 to deduce that F has a unique fixed point in $\overline{B_S(x_0, r_0)} \subset B_S(x_0, r)$. Again, it is easy to see that F has only one fixed point in $B_S(x_0, r)$. ■

Next, we examine briefly the behavior of a contractive map defined on $\overline{B_S(r)} = \overline{B_S(0, r)}$ (the closed ball of radius r with centre 0) with values in Banach space E. More general results will be presented in the next theorem.

THEOREM 3.5. *Let (X, S) be a complete S-metric space with $S(x, y, z) = \|x - y\| + \|y - z\|$ and let $\overline{B_S(r)}$ be the closed ball of radius $r > 0$, central at zero in Banach space E with $F : \overline{B_S(r)} \to E$ a contraction and $F(\partial \overline{B_S(r)}) \subseteq \overline{B_S(r)}$. Then F has a unique fixed point in $\overline{B_S(r)}$.*

Proof. Consider $G(x) = \dfrac{x + F(x)}{2}$. We first show that $G : \overline{B_S(r)} \to \overline{B_S(r)}$. To see this, let

$$x^* = r\frac{x}{\|x\|} \quad \text{where} \quad x \in \overline{B_S(r)} \text{ and } \quad x \neq 0.$$

Now if $x \in \overline{B_S(r)}$ and $x \neq 0$, we have

$$\begin{aligned} S(F(x), F(x), F(x^*)) = \|F(x) - F(x^*)\| &\leq L\ S(x, x, x^*) = L\ \|x - x^*\| \\ &= L\ \|x - r\frac{x}{\|x\|}\| = L\ (r - \|x\|) \end{aligned}$$

Hence

$$\|F(x)\| \leq \|F(x^*)\| + \|F(x) - F(x^*)\| \leq r + L\ (r - \|x\|) < 2r - \|x\|$$

Then for $x \in \overline{B_S(r)}$ and $x \neq 0$

$$\|G(x)\| = \|\frac{x + F(x)}{2}\| \leq \frac{\|x\| + \|F(x)\|}{2} \leq r.$$

In fact by the continuity of G we get $\|G(0)\| \leq r$, and consequently $G : \overline{B_S(r)} \to \overline{B_S(r)}$. Moreover $G : \overline{B_S(r)} \to \overline{B_S(r)}$ is a contraction because

$$\|G(x) - G(y)\| \leq \frac{\|x - y\| + L\|x - y\|}{2} = \frac{(1 + L)}{2}\|x - y\|.$$

Theorem 3.1 implies that G has a unique fixed point in $u \in \overline{B_S(r)}$ and so $u = Fu$. ■

REFERENCES

[**1**] R. Chugh, T. Kadian, A. Rani, B.E. Rhoades, *Property P in G-metric spaces*, Fixed Point Theory Appl. Vol. 2010, Article ID 401684.

[**2**] B.C. Dhage, *Generalized metric spaces mappings with fixed point*, Bull. Calcutta Math. Soc. **84** (1992), 329–336.

[**3**] S. Gähler, *2-metrische Räume und iher topoloische Struktur*, Math. Nachr. **26** (1963), 115–148.

[**4**] Z. Mustafa, B. Sims, *A new approach to generalized metric spaces*, J. Nonlinear Convex Anal. **7** (2006), 289–297.

[**5**] Z. Mustafa, H. Obiedat, F. Awawdeh, *Some common fixed point theorems for mapping on complete G-metric spaces*, Fixed Point Theory Appl. Vol. 2008, Article ID 189870.

[**6**] Z. Mustafa, *A New Structure for Generalized Metric Spaces with Applications to Fixed Point Theory*, Ph. D. Thesis, The University of Newcastle, Callaghan, Australia, 2005.

[**7**] Z. Mustafa, B. Sims, *Some results concerning D-metric spaces*, Proc. Internat. Conf. Fixed Point Theory and Applications, pp. 189–198, Valencia, Spain, 2003.

[**8**] S.V.R. Naidu , K.P.R. Rao, N. Srinivasa Rao, *On the topology of D-metric spaces and the generation of D-metric spaces from metric spaces*, Internat. J. Math. Math. Sci. **2004** (2004), No. 51, 2719–2740.

[**9**] S.V.R. Naidu, K.P.R. Rao, N. Srinivasa Rao, *On the concepts of balls in a D-metric space*, Internat. J. Math. Math. Sci. **2005** (2005), 133–141.

[**10**] S.V.R. Naidu, K.P.R. Rao, N. Srinivasa Rao, *On convergent sequences and fixed point theorems in D-metric spaces*, Internat. J. Math. Math. Sci. **2005** (2005), 1969–1988.

[**11**] S. Sedghi, K.P.R. Rao, N. Shobe, *Common fixed point theorems for six weakly compatible mappings in D^*-metric spaces*, Internat. J. Math. Math. Sci. 6 (2007), 225–237.

[**12**] S. Sedghi, N. Shobe, H. Zhou, *A common fixed point theorem in D^*-metric spaces*, Fixed Point Theory Appl. Vol. 2007, Article ID 27906, 13 pages.

[**13**] W. Shatanawi, *Fixed point theory for contractive mappings satisfying Φ-maps in G-metric spaces*, Fixed Point Theory Appl. Vol. 2010, Article ID 181650.

(received 10.02.2011; in revised form 08.08.2011; available online 10.09.2011)

Shaban Sedghi, Department of Mathematics, Qaemshahr Branch, Islamic Azad University, Qaemshahr, Iran

E-mail: `sedghi_gh@yahoo.com, sedghi.gh@qaemshahriau.ac.ir`

Nabi Shobe, Department of Mathematics, Islamic Azad University-Babol Branch, Iran

E-mail: `nabi_shobe@yahoo.com`

Abdelkrim Aliouche, Department of Mathematics, University of Larbi Ben M'Hidi Oum-El-Bouaghi, 04000, Algeria

E-mail: `alioumath@yahoo.fr`