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ANTINORMAL COMPOSITION OPERATORS ON $l^2(\lambda)$

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Abstract. In this paper we characterize self-adjoint and normal composition operators on Poisson weighted sequence spaces $l^2(\lambda)$. However, the main purpose of this paper is to determine explicit conditions on inducing map under which a composition operator admits a best normal approximation. We extend results of Tripathi and Lal [Antinormal composition operators on l^2, Tamkang J. Math. 39 (2008), 347-352] to characterize antinormal composition operators on $l^2(\lambda)$.

1. Introduction and preliminaries

The distance of an operator to the set of normal operators has been studied in [5, 7, 11]. In [6], Holmes posed the question: Does every operator admit a normal approximation? Holmes pays special emphasis on those operators which admit zero as a best normal approximant. He named such operators as antinormal operators. The same problem has been studied for the first time in context of composition operators on the Hilbert space l^2 in [17] by Tripathi and Lal. The notion of composition operator appeared implicitly in the work of Hardy and Littlewood [9] in 1925. A systematic study of this class of operators was initiated by Ryff [12] and Nordgren [10]. The term composition operator was coined by Nordgren in his paper [10].

Let X be a non-empty set and $V(X)$ be a linear space of complex valued functions on X under pointwise addition and scalar multiplication. If ϕ is a selfmap on X such that composition $f \circ \phi$ belongs to V(X) for each $f \in V(X)$, then ϕ induces a linear transformation on $V(X)$ into itself given by $C_\phi f = f \circ \phi$. The transformation C_ϕ is known as composition transformation. When $V(X)$ is a Banach space or Hilbert space and C_ϕ is a bounded linear operator on $V(X)$, then C_ϕ is called a composition operator.

Monographs [13] and [15] are elegant references for the theory of composition operators. For details on composition operators on l^2 we refer to [14].

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In this paper, $\mathbb{N}_0$ and $\mathbb{C}$ denote the set of all non-negative integers and the set of all complex numbers respectively. Let ϕ be a function on $\mathbb{N}_0$ and $\phi^{-1}(n)$ denote the inverse image of n under ϕ. We denote by $|\phi^{-1}(n)|$ the cardinality of the set $\phi^{-1}(n)$. Also, $\chi_n : \mathbb{N}_0 \to \mathbb{N}_0$ is defined as

$$\chi_n(m) = \begin{cases} 1, & \text{if } m = n, \\ 0, & \text{otherwise.} \end{cases}$$

Let H be a separable complex Hilbert space and let $B(H)$ denote the algebra of all bounded linear operators on H. Further, for $T \in B(H)$, let $N(T)$ and $R(T)$ respectively denote the null space and the range space of T.

Poisson distribution is named after French mathematician Simeon-Denis Poisson, who introduced it in 1837. Poisson distribution with parameter $\lambda > 0$ is defined as $w(n) = e^{-\lambda}\frac{\lambda^n}{n!}$, where $n \in \mathbb{N}_0$. For the details of Poisson distribution we refer to [3].

For $\lambda > 0$, $l^2(\lambda) = \{f : \mathbb{N}_0 \to \mathbb{C} \mid \sum_{n\in\mathbb{N}_0} \frac{e^{-\lambda}\lambda^n}{n!}|f(n)|^2 < \infty\}$ is the Hilbert space of all square summable Poison weighted sequences of complex numbers under the inner product

$$\langle f,\ g\rangle = \sum_{n\in\mathbb{N}_0} f(n)\overline{g(n)}\,\frac{e^{-\lambda}\lambda^n}{n!} \quad \forall f, g \in l^2(\lambda).$$

The following results proved in [8] are relevant to our context.

Theorem 1.1. *A composition transformation C_ϕ is bounded on $l^2(\lambda)$ if and only if there exists a real number $M > 0$ such that*

$$\sum_{m\in\phi^{-1}(n)} \frac{e^{-\lambda}\lambda^m}{m!} \le M\frac{e^{-\lambda}\lambda^n}{n!} \quad \forall n \in \mathbb{N}_0.$$

Theorem 1.2. *Let C_ϕ be a composition operator on $l^2(\lambda)$. Then, C_ϕ is injective if and only if ϕ is surjective.*

Theorem 1.3. *If $f = \sum_{n\in\mathbb{N}_0} f(n)\chi_n \in l^2(\lambda)$, then the adjoint of C_ϕ is $C_\phi^*(f) = \sum_{n\in\mathbb{N}_0} f(n)\xi_n \cdot \chi_{\phi(n)}$, where $\cdot$ denotes pointwise operation and $\xi_n(m) = \frac{\lambda^n}{n!}\frac{m!}{\lambda^m}$ $\forall\ m \in \mathbb{N}_0$.*

Theorem 1.4. *Let C_ϕ be a composition operator on $l^2(\lambda)$. Then, adjoint C_ϕ^* of C_ϕ is injective if and only if ϕ is injective.*

Recall the following definitions and properties.

An operator $T \in B(H)$ is said to be a Fredholm operator if the dimension of $N(T)$ and the dimension of the quotient space $H/R(T))$ are finite. The essential spectrum of an operator T is defined as $\sigma_e(T) = \{\alpha \in \mathbb{C} : T - \alpha I \text{ is not Fredholm}\}$. Since every invertible operator is Fredholm operator, hence $\sigma_e(T) \subseteq \sigma(T)$, see [4].

The minimum modulus of an operator $T \in B(H)$ is defined as $m(T) = \inf\{\|Tx\| : \|x\| = 1\}$, and the essential minimum modulus of an operator $T \in B(H)$ is defined as $m_e(T) = \inf\{\alpha \ge 0 : \alpha \in \sigma_e(|T|)\}$, where $|T| = (T^*T)^{\frac{1}{2}}$.

An operator $T \in B(H)$ is said to be antinormal if $d(T, \mathcal{N}) = \inf_{N \in \mathcal{N}} \|T - N\| = \|T\|$, where $\mathcal{N}$ is class of all normal operators in $B(H)$. T is antinormal if its adjoint T^* is antinormal.

For an operator T in $B(H)$, the index of T is defined as

$$\text{index}(T) = \begin{cases} \dim(N(T)) - \dim(N(T^*)), & \text{if } \dim(N(T)) \text{ or} \\ & \quad \dim(N(T^*)) < \infty, \\ 0, & \text{otherwise.} \end{cases}$$

Observe that $\text{index}(T) = -\,\text{index}(T^*)$.

We now state the following important results proved by Izumino in [7] which we will use later in the paper.

Theorem 1.5. *If* $\text{index}(T) = 0$*, then* $d(T, \mathcal{N}) \leq \frac{\|T\| - m(T)}{2}$.

Corollary 1.1. *If* $\text{index}(T) = 0$*, then* T *cannot be antinormal.*

Theorem 1.6. *If* $\text{index}(T) < 0$*, then* $m_e(T) \leq d(T, \mathcal{N}) \leq \frac{\|T\| + m_e(T)}{2}$.

Corollary 1.2. *If* $\text{index}(T) < 0$*, then* T *is antinormal if and only if* $m_e(T) = \|T\|$.

Let $(X, \mathcal{S}, \mu)$ be a measure space. A measurable set E is called an atom if $\mu(E) \neq 0$ and for each measurable subset F of E either $\mu(F) = 0$ or $\mu(F) = \mu(E)$. A measure space $(X, \mathcal{S}, \mu)$ is called an atomic measure space if each measurable subset of non-zero measure contains an atom.

A trivial example of an atomic measure space is $(X, \mathcal{S}, \mu)$, where X is any non-empty set, $\mathcal{S}$ is a σ-algebra and μ is the counting measure.

An atomic measure space $(X, \mathcal{S}, \mu)$ is called a finite atomic measure space if $\mu(X) < \infty$. In [16], Singh and Veluchamy gave the following characterization. If $(X, \mathcal{S}, \mu)$ is a finite atomic measure space and C_ϕ is a composition operator on $L^2(\mu)$, then the following statements are equivalent: (i) C_ϕ is unitary, (ii) C_ϕ is normal, (iii) C_ϕ is an isometry, (iv) C_ϕ is quasinormal, (v) C_ϕ is a co-isometry.

If the atoms $\{A_n\}_{n=1}^{\infty}$ in the finite measure space $(X, \mathcal{S}, \mu)$ are such that $\mu(A_m) \neq \mu(A_n)$ whenever m and n are different then all above statements (i) to (v) imply that C_ϕ is the identity operator.

2. Main results

2.1. Self-adjoint, normal composition operators on $l^2(\lambda)$

In this section we characterize self-adjoint, normal composition operators.

Theorem 2.1. *A composition operator* C_ϕ *on* $l^2(\lambda)$*, where* $\lambda \neq 1$*, is self-adjoint if and only if* ϕ *is identity. For* $\lambda = 1$*,* C_ϕ *on* $l^2(\lambda)$ *is self-adjoint if and only if* ϕ *is identity or* ϕ *has the following form:*

$$\phi(n) = \begin{cases} 1, & \text{if } n = 0 \\ 0, & \text{if } n = 1 \\ n, & \text{otherwise}. \end{cases}$$

Proof. Suppose C_ϕ is self-adjoint. Then

$$\begin{aligned} C_\phi = C_\phi^* &\iff C_\phi(\chi_n) = C_\phi^*(\chi_n) \quad \forall\, n \in \mathbb{N}_0 \\ &\iff \chi_{\phi^{-1}(n)} = \frac{\lambda^n}{n!}\frac{\phi(n)!}{\lambda^{\phi(n)}}\chi_{\phi(n)} \quad \forall\, n \in \mathbb{N}_0 \\ &\iff \chi_{\phi^{-1}(n)} = \chi_{\phi(n)} \text{ and } \frac{\lambda^n}{n!} = \frac{\lambda^{\phi(n)}}{\phi(n)!} \quad \forall\, n \in \mathbb{N}_0 \\ &\iff \phi \circ \phi = I \text{ and } \frac{\lambda^n}{n!} = \frac{\lambda^{\phi(n)}}{\phi(n)!} \quad \forall\, n \in \mathbb{N}_0 \\ &\iff \phi \circ \phi = I \text{ and } \lambda^{\phi(n)-n} = \frac{\phi(n)!}{n!} \quad \forall\, n \in \mathbb{N}_0. \end{aligned}$$

Now, if $\lambda \neq 1$ and ϕ is not an identity map, then λ vary with n. This is a contradiction. Hence the first assertion. Also, from above equation if $\lambda = 1$, then

$$C_\phi \text{ is self-adjoint } \iff \phi \circ \phi = I \text{ and } n! = \phi(n)! \ \ \forall\, n \in \mathbb{N}_0.$$

Thus second assertion follows immediately. ∎

THEOREM 2.2. *A composition operator C_ϕ on $l^2(\lambda)$ is normal if and only if* $\sum_{m\in\phi^{-1}(n)} \frac{\lambda^m}{m!} = \left(\frac{\lambda^n}{n!}\right)^2 \frac{\phi(n)!}{\lambda^{\phi(n)}}$ $\forall\, n \in \mathbb{N}_0$.

Proof. By definition we have

$$\begin{aligned} C_\phi \textit{ is normal} &\iff \|C_\phi(f)\| = \|C_\phi^*(f)\| \quad \forall\, f \in l^2(\lambda) \\ &\iff \|C_\phi(\chi_n)\| = \|C_\phi^*(\chi_n)\| \quad \forall\, n \in \mathbb{N}_0 \\ &\iff \|\chi_{\phi^{-1}(n)}\|^2 = \|\frac{\lambda^n}{n!}\frac{\phi(n)!}{\lambda^{\phi(n)}}\chi_{\phi(n)}\|^2 \quad \forall\, n \in \mathbb{N}_0 \\ &\iff \sum_{m\in\phi^{-1}(n)} \frac{\lambda^m}{m!} = \left(\frac{\lambda^n}{n!}\right)^2 \frac{\phi(n)!}{\lambda^{\phi(n)}} \quad \forall\, n \in \mathbb{N}_0. \end{aligned}$$

Hence the proof. ∎

The following remark shows a connection between normal composition operators and invertible composition operators.

REMARK 2.1. If we take $X = \mathbb{N}_0$, $A_n = \{n\}$ and $\mu(A_n) = e^{-\lambda}\frac{\lambda^n}{n!}$, where $n \in \mathbb{N}_0$. Then it follows readily that $L^2(\mu) = l^2(\lambda)$ where μ is finite atomic measure. Hence C_ϕ is normal if and only if ϕ is identity. This implies that every normal operator is invertible.

REMARK 2.2. It is interesting to note that normal composition operators and invertible composition operators are equivalent on l^2.

The following example shows that this not true in $l^2(\lambda)$ for $\lambda \neq 1$.

EXAMPLE 2.1. Let $\phi\colon \mathbb{N}_0 \to \mathbb{N}_0$ be defined as

$$\phi(n) = \begin{cases} 1, & \text{if } n = 0 \\ 0, & \text{if } n = 1 \\ n, & \text{otherwise} . \end{cases}$$

Then $C_\phi^* C_\phi f = f(0)\chi_0 + \frac{f(1)}{\lambda} f(1)\chi_1 + \sum_{n\geq 2} f(n)\chi_n$ and $C_\phi C_\phi^* f = \frac{f(0)}{\lambda} f(0)\chi_0 + f(1)\chi_1 + \sum_{n\geq 2} f(n)\chi_n$. Hence C_ϕ is not normal until $\lambda = 1$. Moreover, Radon-Nikodym derivative of $\mu\phi^{-1}$ with respect to μ, denoted by f_ϕ, is as follows:

$$f_\phi = \lambda\chi_0 + \frac{1}{\lambda} f(1)\chi_1 + \sum_{n\geq 2} \chi_n .$$

Clearly f_ϕ is bounded away from zero and range of C_ϕ is $l^2(\lambda)$. Hence by [15, Theorem 2.2.11] C_ϕ is invertible. ∎

2.2. Antinormal composition operators on $l^2(\lambda)$

In [17], Tripathi and Lal have characterized antinormal composition operators on sequence space l^2. Now we state this characterization as follows.

THEOREM 2.3. *Let C_ϕ be a composition operator on l^2.*

(i) *If ϕ is bijective then C_ϕ is not antinormal.*

(ii) *If ϕ is injective but not surjective then C_ϕ is antinormal.*

(iii) *ϕ is surjective but not injective then C_ϕ is antinormal if and only if $|\phi^{-1}(n)| = \|C_\phi\|^2$ for all but finitely many $n \in \mathbb{N}$.*

(iv) *Suppose ϕ is neither injective nor surjective.*
   (a) *If $index(C_\phi) < 0$, C_ϕ is antinormal if and only if $|\phi^{-1}(n)| = \|C_\phi\|^2$ for all but finitely many $n \in \mathbb{N}$.*
   (b) *If $index(C_\phi) \geq 0$, C_ϕ is not antinormal.*

Now we cite two examples of antinormal composition operators on l^2.

EXAMPLE 2.2. The function ϕ on $\mathbb{N}$ into itself defined by $\phi(n) = n + 1$ is injective but not surjective. The composition operator C_ϕ is antinormal by case (ii).

EXAMPLE 2.3. The function ϕ on $\mathbb{N}$ into itself defined by

$$\phi(n) = \begin{cases} n, & \text{if } n = 1, 2 \\ \frac{n+3}{2}, & \text{if } n \geq 3 \text{ and } n \text{ is odd} \\ \frac{n}{2} + 1, & \text{if } n \geq 4 \text{ and } n \text{ is even.} \end{cases}$$

is surjective but not injective. The composition operator C_ϕ is antinormal by case (iii) since $|\phi^{-1}(n)| = 2$ for all $n \in \mathbb{N}$, except $n = 1$.

Motivated by the above results we investigate antinormal composition operators on $l^2(\lambda)$ in terms of inducing map in the following cases.

(i) ϕ is bijective.

(ii) ϕ is not bijective then following cases are possible.
   (a) ϕ is injective but not surjective.
   (b) ϕ is surjective but not injective.
   (c) ϕ is neither injective nor surjective.

Remark 2.3. If ϕ is bijective then C_ϕ and C_ϕ^* both are injective by Theorems 1.2 and 1.4, respectively. Therefore index$(C_\phi) = 0$. Hence C_ϕ is not antinormal.

Theorem 2.4. *Suppose ϕ is injective but not surjective. If ϕ is such that no term of the sequence $\{\frac{\lambda^n}{n!}\frac{\phi(n)!}{\lambda^{\phi(n)}}\}$ repeats itself infinitely many times then C_ϕ is not antinormal.*

Proof. It suffices to prove that C_ϕ^* is not antinormal. Let $\alpha \geq 0$. Then for each $f \in l^2(\lambda)$ we get

$$\begin{aligned}(C_\phi C_\phi^* - \alpha I)f &= (C_\phi C_\phi^* - \alpha I)\sum_{n\in\mathbb{N}_0} f(n)\chi_n \\ &= \sum_{n\in\mathbb{N}_0}\Big(\frac{\lambda^n}{n!}\frac{\phi(n)!}{\lambda^{\phi(n)}} - \alpha\Big)f(n)\chi_n.\end{aligned}$$

By our assumption, the factor $\left(\frac{\lambda^n}{n!}\frac{\phi(n)!}{\lambda^{\phi(n)}} - \alpha\right)$ can not be zero for infinitely many $n's$ in $\mathbb{N}_0$. This implies $C_\phi C_\phi^* - \alpha I$ is Fredholm for each $\alpha \geq 0$. Hence $m_e(C_\phi^*) = \infty$. Also, the index$(C_\phi^*) < 0$ as C_ϕ^* is injective. Now as $m_e(C_\phi^*) \neq \|C_\phi\|$ and index$(C_\phi^*) < 0$, hence C_ϕ^* by Corollary 1.2 is not antinormal. Consequently C_ϕ is not antinormal. ■

Our next theorem gives a sufficient condition for C_ϕ to be antinormal on $l^2(\lambda)$.

Theorem 2.5. *Suppose ϕ is surjective but not injective. If the set $\{n \in \mathbb{N}_0 : \sum_{m\in\phi^{-1}(n)}\frac{\lambda^m}{m!}\frac{n!}{\lambda^n} = \alpha\}$ is finite for every $\alpha < \|C_\phi\|^2$ and the set $\{n \in \mathbb{N}_0 : \sum_{m\in\phi^{-1}(n)}\frac{\lambda^m}{m!}\frac{n!}{\lambda^n} = \|C_\phi\|^2\}$ is infinite, then C_ϕ is antinormal.*

Proof. Performing simple computation we get

$$(C_\phi^* C_\phi - \alpha I)f = \sum_{n\in\mathbb{N}_0}\Big(\sum_{m\in\phi^{-1}(n)}\frac{\lambda^m}{m!}\frac{n!}{\lambda^n} - \alpha\Big)f(n)\chi_n.$$

For $0 \leq \alpha < \|C_\phi\|^2$, $\dim(N(C_\phi^*C_\phi - \alpha I)) = \dim(N(C_\phi^*C_\phi - \alpha I)^*)$ is finite by our assumption. Hence $\alpha \notin \sigma_e(|C_\phi|)$ for $0 \leq \alpha < \|C_\phi\|$. But for $\alpha = \|C_\phi\|^2$, $\dim N((C_\phi^*C_\phi - \alpha I))$ is infinite, by the given assumption. Hence $\|C_\phi\| \in \sigma_e(|C_\phi|)$. Thus $m_e(C_\phi) = \|C_\phi\|$. Also, the index$(C_\phi) < 0$ as C_ϕ is injective. Hence C_ϕ is antinormal. ■

The following result gives a necessary condition for C_ϕ to be antinormal.

Theorem 2.6. *Suppose ϕ is surjective but not injective. If C_ϕ is antinormal then set $\{n \in \mathbb{N}_0 : \sum_{m\in\phi^{-1}(n)}\frac{\lambda^m}{m!}\frac{n!}{\lambda^n} = \alpha\}$ is finite for every $\alpha < \|C_\phi\|^2$.*

Proof. On the contrary, assume that there exists a positive real number α_0 such that the set $\{n \in \mathbb{N}_0 : \sum_{m\in\phi^{-1}(n)} \frac{\lambda^m}{m!}\frac{n!}{\lambda^n} = \alpha_0 < \|C_\phi\|^2\}$ is infinite. Now consider the following equation

$$(C_\phi^* C_\phi - \alpha_0 I)f = \sum_{n\in\mathbb{N}_0} \Big(\sum_{m\in\phi^{-1}(n)} \frac{\lambda^m}{m!}\frac{n!}{\lambda^n} - \alpha_0 \Big) f(n)\chi_n.$$

The above equation shows that $\dim(N(C_\phi^* C_\phi - \alpha_0 I))$ is infinite. Hence $C_\phi^* C_\phi - \alpha_0 I$ is not Fredholm. Hence $\sqrt{\alpha_0} \in \sigma_e(|C_\phi|)$. This implies

$$m_e(C_\phi) \le \sqrt{\alpha_0} < \|C_\phi\|.$$

Again observe that index$(C_\phi) < 0$, as C_ϕ is injective. Hence C_ϕ is not antinormal. ∎

Before exploring the case when ϕ is neither injective nor surjective, we prove the following lemma.

Lemma 2.1. $\dim(N(C_\phi^*)) = \sum_{n\in\mathbb{N}_0} \Big(|\phi^{-1}(n)| - 1\Big)$.

Proof. We first show that $\dim(R(C_\phi))^\perp = \sum_{n\in\mathbb{N}_0} |\phi^{-1}(n)| - 1$. Let

$$\begin{aligned} f \in R(C_\phi)^\perp &\iff \langle f, g\rangle = 0 \ \forall g \in R(C_\phi) \\ &\iff \langle f, C_\phi h\rangle = 0 \ \forall h \in l^2(\lambda) \\ &\iff \langle C_\phi^* f, h\rangle = 0 \ \forall h \in l^2(\lambda) \\ &\iff \langle C_\phi^* f, \chi_n\rangle = 0 \ \forall \chi_n \in l^2(\lambda) \\ &\iff \sum_{n\in\mathbb{N}_0} \Big(\sum_{m\in\phi^{-1}(n)} f(m)\frac{\lambda^m}{m!}\Big) \frac{\phi(n)!}{\lambda^{\phi}(n)}\chi_n = 0 \ \forall n \in \mathbb{N}_0 \\ &\iff \sum_{m\in\phi^{-1}(n)} f(m)\frac{\lambda^m}{m!} = 0 \ \forall n \in \mathbb{N}_0. \end{aligned}$$

This implies that $\dim(R(C_\phi))^\perp = \sum_{n\in\mathbb{N}_0} \Big(|\phi^{-1}(n)| - 1\Big)$. Further since $N(C_\phi^*) = (R(C_\phi))^\perp$, hence the result follows. ∎

Theorem 2.7. *Suppose ϕ is neither injective nor surjective.*

(a) *If the* index$(C_\phi) > 0$ *and no term of the sequence* $\{\frac{\lambda^n}{n!}\frac{\phi(n)!}{\lambda^{\phi(n)}}\}$ *repeats infinitely many times, then C_ϕ is not antinormal.*

(b) *If the* index$(C_\phi) < 0$*, the set* $\{n \in \mathbb{N}_0 : \sum_{m\in\phi^{-1}(n)} \frac{\lambda^m}{m!}\frac{n!}{\lambda^n} = \alpha\}$ *is finite for every $\alpha < \|C_\phi\|^2$ and the set* $\{n \in \mathbb{N}_0 : \sum_{m\in\phi^{-1}(n)} \frac{\lambda^m}{m!}\frac{n!}{\lambda^n} = \|C_\phi\|^2\}$ *is infinite, then C_ϕ is antinormal.*

(c) *If the* index$(C_\phi) = 0$*, then C_ϕ is not antinormal.*

Proof. If index$(C_\phi) > 0$ then index$(C_\phi^*) < 0$. Therefore $\dim(N(C_\phi^*))$ is finite. Hence by Lemma 2.1 $|\phi^{-1}(n)| = 1$ for all but finitely many $n \in \mathbb{N}_0$. Now using the arguments used in Theorem 2.4, $\sigma_e(|C_\phi^*|) = \emptyset$. Consequently $m_e(C_\phi^*) = \infty$. Therefore C_ϕ is not antinormal. The result (b) is immediate from the Theorem 2.5. Part (c) follows from the Corollary 1.1. ■

EXAMPLE 2.4. The function ϕ on $\mathbb{N}_0$ into itself defined by $\phi(n) = n + 1$ is injective but not surjective. It is easy to see that no term of the sequence $\frac{\lambda^n}{n!}\frac{\phi(n)!}{\lambda^{\phi(n)}}$ repeats infinitely many times. Consequently, C_ϕ is not antinormal by Theorem 2.4.

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