MATEMATIČKI VESNIK
МАТЕМАТИЧКИ ВЕСНИК
78, 1 (2026), 83–94
March 2026

research paper
оригинални научни рад
DOI: 10.57016/MV-RYLe8813

WEIGHTED CHRIS-JERRY DISTRIBUTION: PROPERTIES AND ITS APPLICATION

C. Subramanian, M. Subhashree, Aafaq A. Rather and Shahid Ahmad Wani

Abstract. In this paper, we have proposed a new extension of the Chris- Jerry distribution called as Weighted Chris- Jerry Distribution which depends on two parameters. The statistical properties of this distribution such as moments, survival functions and hazard rate, reverse hazard rate, order statistics, entropies and likelihood ratio test are derived. The two parameters are estimated using maximum likelihood estimator. To illustrate this distribution, real life data set is considered. This data set is analyzed through this distribution, to show how the proposed model worked in it.

1. Introduction

The concept of weighted distribution has become popular in modeling lifetime data. It was first proposed by Sir Ronald Aylmer Fisher (1934). The idea of weighted distributions can be traced to the study of "the effect of methods of ascertainment upon estimation of frequencies." Later, it was developed by Rao (1965). Weighted distribution has been applied to various designs of scientific experiments for the development of statistical models. It is also widely applied in various research fields such as medicine, reliability, ecology, behavioral sciences, finance, insurance, etc. It provides a different perspective on the existing standard probability distributions. The weighted distribution is unique in providing flexibility. It also has applications in modeling lifetime data, cluster sampling, and heterogeneity. Overall, weighted distribution sets a milestone in modeling statistical data efficiently and precisely. It performs well in cases where the standard distribution fails to do so.

The concept of length-bias was first suggested by Cox [1] and Zelen [15]. Van Deusen [14] independently acknowledged the size-biased distribution theory. Generally, the size-biased distribution occurs when the sampling mechanism selects units with probability proportional to some measure of the unit size.

2020 Mathematics Subject Classification: 62F03, 62N05, 62F12, 62G30, 62E15.
Keywords and phrases: Weighted distribution; Chris- Jerry distribution; reliability analysis; maximum likelihood estimator; order statistics; likelihood ratio test.

The Lindley distribution, proposed by Ghitany, Atieh, and Nadarajah [4], is formed using an exponential distribution with scale parameter θ and a gamma distribution having shape and scale parameters θ, where $p = \frac{\theta}{\theta+1}$. The statistical properties such as moments, reliability analysis, maximum likelihood estimation of parameters, and entropies are discussed. Gupta and Kundu [5] introduced a weighted distribution as a new model whose probability density function (pdf) closely resembles the pdfs of the Weibull, Gamma, and generalized exponential distributions. The analysis of statistical properties for various samples has been demonstrated.

The combination of exponential, Weibull, exponentiated-Weibull, gamma, exponentiated-exponential, and exponentiated-exponential distributions forms the gamma exponentiated-Weibull distribution proposed by Pinho, Cordeiro and Nobre [6]. It is used for failure time data analysis. Sen, Maiti, and Chandra [11] proposed the X-Gamma distribution as a special finite mixture of exponential and gamma distributions. It has applications in the analysis of real lifetime datasets on the remission times of patients receiving an analgesic.

The two-parameter weighted Shanker distribution was introduced by Shanker and Shukla [13]. It is demonstrated using real lifetime datasets, and the fit is found to be quite satisfactory when compared with other distributions. The weighted length-biased generalized uniform distribution is a new extension of the generalized uniform distribution proposed by Rather and Subramanian [10]. The weighted Pareto Type II distribution was introduced by Para and Jan [7]. It is used for handling medical science data, and its properties have been studied. The model combining two distributions – the Poisson-weighted Lindley distribution – was introduced by Shanker and Shukla [12] and was found to provide a better fit compared to other distributions. A new exponentiated Garima distribution with scale and shape parameters was introduced by Rather and Subramanian [9], which has been applied to real lifetime datasets effectively.

2. Chris-Jerry distribution (CJD)

The scale parameter θ and the shape parameter of the exponential distribution and Gamma distribution are used to form a new one parameter lifetime distribution called the Chris- Jerry distribution introduced by Chrjaggeysa and Okechukwu [2]. Then, the pdf and cdf given by

$$f_{Cj}(x,\theta) = \frac{\theta^2}{\theta+2}\left(1+\theta x^2\right)e^{-\theta x}; x>0, \theta>0, \tag{1}$$

$$F_{Cj}(x,\theta) = 1-\left[1+\frac{\theta x(\theta x+2)}{\theta+2}\right]e^{-\theta x}; x>0, \theta>0. \tag{2}$$

3. Weighted Chris-Jerry distribution

Let X be a non-negative random variable and $w(X)$ a non-negative weight function. Then, the pdf of the weighted random variable X_w is given by

$$f_w(X) = \frac{w(X)f(x)}{E(w(X))}; x > 0, \tag{3}$$

where $w(X)$ is a non-negative weight function and $E(w(X)) = \int w(X)f(x)dx < \infty$.

In this paper, we consider the weight function $w(X) = X^c$ to obtain the CJD. The function $w(X) = X^c$ is a common choice because it allows for flexible adjustments based on the value of X, is mathematically simple, and often facilitates easier computations, making it a convenient choice in various statistical analyses.

Then, the pdf of the weighted Chris-Jerry distribution (WCJD) is given by

$$f_w(X) = \frac{x^c f(x)}{E\left(X^c\right)}; x > 0. \tag{4}$$

Here, $E\left(X^c\right) = \int_0^\infty x^c f(x:\theta)dx$

$$E\left(X^c\right) = \frac{(\theta\Gamma(c+1) + \Gamma(c+3))}{\theta^c(\theta+2)}. \tag{5}$$

Here (Γ) is a gamma function and has been used in the above equation and can be defined as

$$\Gamma(x) = \int_0^\infty e^{-\theta}\theta^{x-1}dx.$$

Substituting (1) and (5) in equation (4), the required pdf of weighted Chris-Jerry distribution (WCJD) is

$$f_{wc}(x) = \frac{\theta^{c+2}}{(\theta\Gamma(c+1) + \Gamma(c+3))} x^c \left(1 + \theta x^2\right) e^{-\theta x}. \tag{6}$$

Then, the cumulative density function (cdf) of the weighted Chris-Jerry distribution (WCJD) is derived as

$$F_w(X) = \int_0^x f_w(X)dx,$$

$$F_w(X) = \int_0^x \frac{\theta^{c+2}}{(\theta\Gamma(c+1) + \Gamma(c+3))} x^c \left(1 + \theta x^2\right) e^{-\theta x}dx.$$

Simplifying the above equation by using incomplete gamma function (γ) which is a related function that involves integration over a specified range. The incomplete gamma function can be defined as

$$\gamma(s, x) = \int_0^x e^{-y}\theta^{x-1}d\theta.$$

Therefore, the cdf of the weighted Chris-Jerry distribution (WCJD) can be

$$F_{wc}(x) = \frac{(\theta\gamma(c+1, \theta x) + \gamma(c+3, \theta x))}{(\theta\Gamma(c+1) + \Gamma(c+3))}. \tag{7}$$

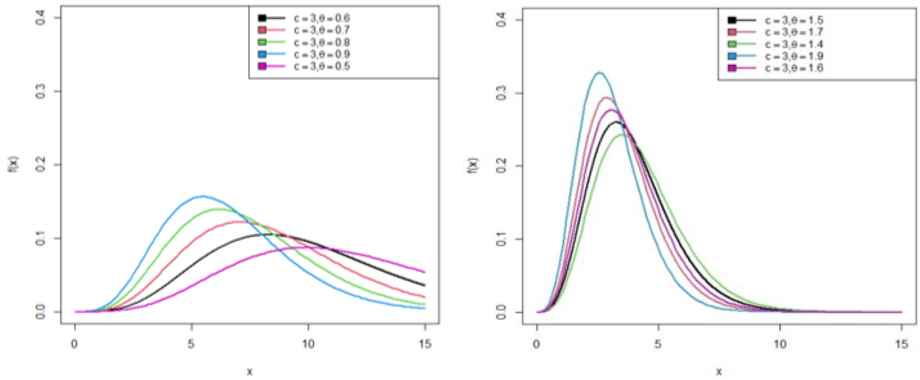

Figure 1: Pdf plot of weighted Chris-Jerry distribution

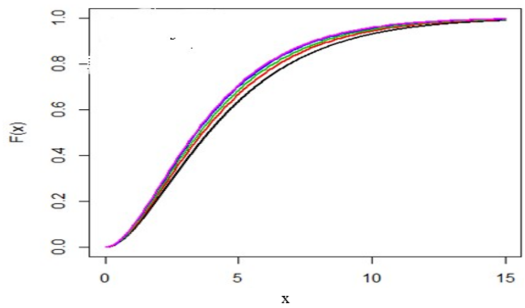

Figure 2: Cdf plot of weighted Chris-Jerry distribution

4. Reliability analysis

In this part, we will explain about the survival function, failure rate, reverse hazard rate and the Mills ratio of the weighted Chris-Jerry distribution.

The survival function or the reliability of the weighted Chris-Jerry distribution is given by

$$S_w(X) = 1 - \frac{(\theta\gamma(c+1,\theta x) + \gamma(c+3,\theta x))}{(\theta\Gamma(c+1) + \Gamma(c+3))}.$$

The hazard function is also known as the hazard rate, instantaneous failure rate or force of mortality and is given by

$$h(x) = \frac{f_w(X)}{1 - F_w(X)},$$

$$h(x) = \frac{\theta^{c+2} x^c \left(1 + \theta x^2\right) e^{-\theta x}}{(\theta\Gamma(c+1) + \Gamma(c+3)) - (\theta\gamma(c+1,\theta x) + \gamma(c+3,\theta x))}.$$

The Reverse hazard rate is given by

$$h_r(x) = \frac{\theta^{c+2} x^c \left(1+\theta x^2\right) e^{-\theta x}}{\left(\theta\gamma(c+1,\theta x)+\gamma(c+3,\theta x)\right)}.$$

The Mills ratio of the weighted Chris-Jerry distribution is

$$\text{Mills Ratio } = \frac{1}{h_r(x)} = \frac{\left(\theta\gamma(c+1,\theta x)+\gamma(c+3,\theta x)\right)}{\theta^{c+2} x^c \left(1+\theta x^2\right) e^{-\theta x}}.$$

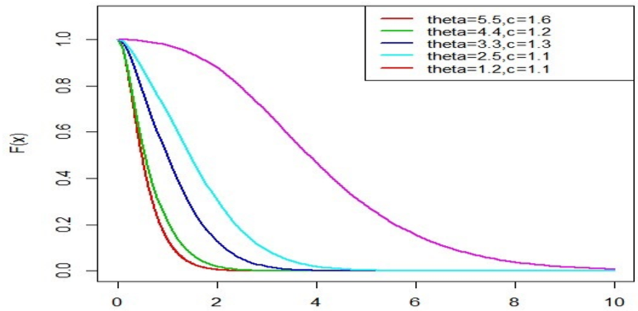

Figure 3: Survival function of Chris-Jerry distribution

5. Moments and related measures

Let X indicate the random variable of weighted Chris-Jerry distribution with parameters θ and c. Then the r^{th} order moment $\mathrm{E}\left(X^r\right)$ of weighted Chris-Jerry distribution is derived as

$$E\left(X^r\right) = \mu_r^j = \int_0^\infty x^r f_w(X),$$

$$\mathrm{E}\left(X^r\right) = \int_0^\infty \frac{\theta^{c+2} x^{c+r}\left(1+\theta x^2\right) e^{-\theta x}}{\left(\theta\Gamma(c+1)+\Gamma(c+3)\right)} dx,$$

$$\mathrm{E}\left(X^r\right) = \frac{\theta^{c+2}}{\left(\theta\Gamma(c+1)+\Gamma(c+3)\right)} \int_0^\infty x^{c+r}\left(1+\theta x^2\right) e^{-\theta x} dx,$$

$$E\left(X^r\right) = \frac{\left(\theta\Gamma(c+r+1)+\Gamma(c+r+3)\right)}{\theta^r\left(\theta\Gamma(c+1)+\Gamma(c+3)\right)}.$$

In equation (8), when $r=1$, the mean of weighted Chris-Jerry distribution which is given by

$$E(X) = \mu_1^j = \frac{\left(\theta\Gamma(c+2)+\Gamma(c+4)\right)}{\theta\left(\theta\Gamma(c+1)+\Gamma(c+3)\right)}. \quad (8)$$

Similarly, when $r=2$, the mean of weighted Chris-Jerry distribution which is given by

$$E\left(X^2\right)=\mu_2^j=\frac{(\theta\Gamma(c+3)+\Gamma(c+5))}{\theta^2(\theta\Gamma(c+1)+\Gamma(c+3))}.$$

Therefore,

$$\text{Variance }=\left\{\frac{(\theta\Gamma(c+3)+\Gamma(c+5)}{\theta^2(\theta\Gamma(c+1)+\Gamma(c+3))}-\left[\frac{(\theta\Gamma(c+2)+\Gamma(c+4))}{\theta(\theta\Gamma(c+1)+\Gamma(c+3))}\right]^2\right\}.$$

5.1 Harmonic mean

The Harmonic mean of the aspired model can be derived as

$$H.M=E\left(\frac{1}{X}\right)=\int_0^\infty\frac{1}{x}f_w(X)dx=\int_0^\infty\frac{1}{x}\frac{\theta^{c+2}}{(\theta\Gamma(c+1)+\Gamma(c+3))}x^c\left(1+\theta x^2\right)e^{-\theta x}dx,$$

$$=\int_0^\infty\frac{1}{x}\frac{\theta^{c+2}}{(\theta\Gamma(c+1)+\Gamma(c+3))}x^c\left(1+\theta x^2\right)e^{-\theta x}dx,$$

$$=\frac{\theta^{c+2}}{(\theta\Gamma(c+1)+\Gamma(c+3))}\int_0^\infty x^{c-1}\left(1+\theta x^2\right)e^{-\theta x}dx,$$

$$H.M=\frac{\theta[\theta\Gamma(c)+\Gamma(c+2)]}{[\theta\Gamma(c+1)+\Gamma(c+3)]}.$$

5.2 Moment generating function and characteristics function of WCJD

Assume X has a weighted Chris-Jerry distribution; we will get the moment-generating function of X as

$$M_X(t)=E\left(e^{tX}\right)=\int_0^\infty e^{tx}f_w(X)dx.$$

Using Taylor's series, we find

$$M_X(t)=E\left(e^{tX}\right)=\int_0^\infty\sum_{j=0}^\infty\frac{t^j}{j!}x^jf_w(X)dx=\sum_{j=0}^\infty\frac{t^j}{j!}\mu_j^j,$$

$$=\sum_{j=0}^\infty\frac{t^j}{j!}\frac{[\theta\Gamma(c+j+1)+\Gamma(c+j+3)]}{\theta^j[\theta\Gamma(c+1)+\Gamma(c+3)]},$$

$$M_x(t)=\frac{1}{[\theta\Gamma(c+1)+\Gamma(c+3)]}\sum_{j=0}^\infty\frac{t^j}{j!}[\theta\Gamma(c+j+1)+\Gamma(c+j+3)].$$

In the game way, we will get the characteristics function of weighted Chris-Jerry distribution, which can be obtained as

$$\varphi_X(t)=E\left(\theta^{itX}\right),$$

$$\varphi_X(it)=\frac{1}{[\theta\Gamma(c+1)+\Gamma(c+3)]}\sum_{j=0}^\infty\frac{it^j}{j!\theta^j}[\theta\Gamma(c+j+1)+\Gamma(c+j+3)].$$

6. Order statistics

Let $X_1, X_2, \cdots X_n$ be the random variable drawn from the continuous population. Their pdf be $f_x(x)$ and cumulative density function with $F_x(x)$. Then, assume $X_{(1}, X_{(2)}, \cdots, X_{(n)}$ be the order statistics of a random sample. Thus, the probability density function of r^{th} order statistics $X_{(r)}$ is given by

$$f_{x(r)}(x) = \frac{n!}{(r-1)!(n-r)!} f_x(x) \left[F_x(x)\right]^{r-1} \left[1 - F_x(x)\right]^{n-r}. \tag{9}$$

Putting the equation (6) and (7) in equation (9), the probability density function of r^{th} order statistics $X_{(r)}$ of weighted Chris-Jerry distribution is given by

$$f_{x(r)}(x) = \frac{n!}{(r-1)!(n-r)!} \left[\frac{\theta^{c+2} x^c \left(1 + \theta x^2\right) e^{-\theta x}}{(\theta\Gamma(c+1) + (c+3)}\right] \times \left[\frac{(\theta\gamma(c+1, \theta x) + \gamma(c+3, \theta x))}{(\theta\Gamma(c+1) + \Gamma(c+3))}\right]^{r-1}$$
$$\times \left[1 - \frac{(\theta\gamma(c+1, \theta x) + \gamma(c+3, \theta x))}{(\theta\Gamma(c+1) + \Gamma(c+3))}\right]^{n-r}.$$

Then, the probability density function of higher order statistics $X_{(n)}$ an be derived as

$$f_{x(n)}(x) = \left[\frac{n\theta^{c+2} x^c \left(1 + \theta x^2\right) e^{-\theta x}}{(\theta\Gamma(c+1) + (c+3)}\right] \times \left[\frac{(\theta\gamma(c+1, \theta x) + \gamma(c+3, \theta x))}{(\theta\Gamma(c+1) + \Gamma(c+3))}\right]^{n-1}.$$

Hence, the probability density function of 1^{st} order statistics $X_{(1)}$ can be obtained as

$$f_{x(1)}(x) = \left[\frac{n\theta^{c+2} x^c \left(1 + \theta x^2\right) e^{-\theta x}}{(\theta\Gamma(c+1) + \Gamma(c+3)}\right] \times \left[1 - \frac{(\theta\gamma(c+1, \theta x) + \gamma(c+3, \theta x))}{(\theta\Gamma(c+1) + \Gamma(c+3))}\right]^{n-1}.$$

7. Maximum likelihood estimator and Fisher information matrix

It is one of the best methods for estimating the parameters of the distribution compared with the other estimating methods. Let $X_t, X_{(2)} \ldots, X_{(n)}$ be the random sample of size n drawn from the Weighted Chris-Jerry distribution, then the likelihood function of Weighted Chris - Jerry distribution is given as:

$$L(x; c, \theta) = \prod_{i=1}^{n} f_w(x; c, \theta) = \prod_{i=1}^{n} \left\{\frac{\theta^{c+2} x_i^c \left(1 + \theta x_i^2\right) e^{-\theta x_i}}{[\theta\Gamma(c+1) + \Gamma(c+3)]}\right\},$$

$$L(x; c, \theta) = \frac{\theta^{n(c+2)}}{[\theta\Gamma(c+1) + \Gamma(c+3)]^n} \prod_{i=1}^{n} x_i^c \left(1 + \theta x_i^2\right) e^{-\theta x_i}.$$

The log likelihood function is

$$\log L = n(c+2)\log\theta - n\log(\theta\Gamma(c+1) + \Gamma(c+3))$$
$$+ c\sum_{i=0}^{n} \log x_i + \sum_{i=0}^{n} \log\left(1 + \theta x_i^2\right) - \theta\sum_{i=0}^{n} x_i. \tag{10}$$

By differentiating equation (10) with respect to θ and c, the maximum likelihood estimates of θ and c can be attained.

$$\frac{\partial \log L}{\partial \theta} = \frac{n(c+2)}{\theta} - \frac{n[\theta\Gamma(c+1)]}{[\theta\Gamma(c+1)+\Gamma(c+3)]} + \sum_{i=1}^{n} \frac{x_i^2}{(1+\theta x_i^2)} - \sum_{i=1}^{n} x_i = 0,$$

$$\frac{\partial \log L}{\partial c} = n\log\theta - n\Psi(c+1) + \sum_{i=1}^{n} \log x_i = 0,$$

where $\Psi(.\ )$ is the digamma function. As the likelihood equations are in complicated form, it is not easy to solve the system of non - linear equations. Henceforth to estimate the required parameters, we are using R and wolfram mathematics. We are using the asymptotic nomality tests to drive the confidence interval If $\hat{\lambda} = (\hat{\theta}, \hat{c})$ indicates the Maximum likelihood estimates of $\lambda = (\theta, c)$, then, $\sqrt{n}(\hat{\lambda} - \lambda) \to N_2\left(0, I^{-1}(\lambda)\right)$. Here I($\lambda$) is Fisher's Information Matrix, i.e.,

$$\mathrm{I}(\lambda) = -\frac{1}{n}\begin{Bmatrix} E\left(\frac{\partial^2 \log L}{\partial e^2}\right) & E\left(\frac{\partial^2 \log L}{\partial\theta\partial c}\right) \\ E\left(\frac{\partial^2 \log L}{\partial\theta\partial\theta}\right) & E\left(\frac{\partial^2 \log L}{\partial c^2}\right) \end{Bmatrix}.$$

Here,

$$E\left(\frac{\partial^2 \log L}{\partial\theta^2}\right) = -\frac{n(c+2)}{\theta^2} - n\left[\frac{\Gamma(c+1)[\theta\Gamma(c+1)]}{[\theta\Gamma(c+1)+\Gamma(c+3)]^2}\right] - \sum_{i=1}^{n} \frac{x_i}{\left(1+\theta x_i^2\right)^2},$$

$$E\left(\frac{\theta^2 \log L}{\partial c^2}\right) = -n\varphi^j(c+1), \quad E\left(\frac{\partial^2 \log L}{\partial\theta\partial c}\right) = \frac{n}{\theta},$$

where $\varphi(.)^j$ is the first order derivative of digamma function. As λ being unknown, $I^{-1}(\lambda)$ is estimated by $I^{-1}(\lambda)$ and this can be used to obtain asymptotic confidence intervals for θ and c.

8. Likelihood ratio test

Suppose $X_{(1)}, X_{(2)}, \cdots, X_{(n)}$ be a random sample from the weighted Chris-Jerry distribution. This hypothesis is tested by

$$H_0 : f(x) = f(x;\theta) \quad \text{against} \quad H_1 = f(x) = f_w(x;c,\theta).$$

To check whether the random sample of size n comes from the Chris - Jerry distribution or Weighted Chris - Jerry distribution, we are using this test statistic

$$\Delta = \frac{L_1}{L_0} = \prod_{i=1}^{n} \frac{f_w(x;c,\theta)}{f(x;\theta)}, \qquad \Delta = \left\{\frac{\theta^c(\theta+2)}{[\theta\Gamma(c+1)+\Gamma(c+3)}\right\}^n \prod_{i=1}^{n} x_i^c.$$

Therefore, the null hypothesis is rejected if

$$\Delta = \left\{\frac{\theta^c(\theta+2)}{[\theta\Gamma(c+1)+\Gamma(c+3)]}\right\}^n \prod_{i=1}^{n} x_i^c > k,$$

$$\Delta^* = \prod_{i=1}^{n} x_i^c > k\left(\frac{[\theta\Gamma(c+1)+\Gamma(c+3)]}{\theta^c(\theta+2)}\right)^n,$$

$$\Delta^* = \prod_{i=1}^{n} x_i^c > k^*,$$

where,
$$k^* = k\left(\frac{[\theta\Gamma(c+1)+\Gamma(c+3)]}{\theta^c(\theta+2)}\right)^n.$$

We can conclude, for large sample size $n, 2\log$ is distributed as chi-square distribution with one degree of freedom. Also, p-value is attained from the chi-square distribution. If $\mathrm{p}\left(\Delta^* > k^*\right)$, where $k^* = \prod_{i=1}^{n} x_i^c$ is less than the specified level of gignificance and $\prod_{i=1}^{n} x_i^c$ is the observed value of the statistic Δ^*, then, reject mull hypothesis.

9. Bonferroni and Lorenz curves

The inequality across a population is measured using Bonferroni and Lorenz curves. It gives a clear graphical representation of income or wealth inequality. It is exclusively important in economics. The Bonferroni and the Lorenz curves are given by

$$B(p) = \frac{1}{p\mu_1^{'}}\int_0^q xf(x)dx,$$

and
$$L(p) = \frac{1}{\mu_1^{'}}\int_0^q xf(x)dx,$$

where,
$$\mu_1^{'} = \frac{[\theta\Gamma(c+2)+\Gamma(c+4)]}{[\theta\Gamma(c+1)+\Gamma(c+3)]},$$

and
$$q = F^{-1}(p),$$

$$B(p) = \frac{\theta[\theta\Gamma(c+1)+\Gamma(c+3)]}{p[\theta\Gamma(c+2)+\Gamma(c+4)]}\int_0^q \frac{\theta^{c+2}}{[\theta\Gamma(c+1)+\Gamma(c+3)]} x^{c+1}\left(1+\theta x^2\right)e^{-\theta x}dx.$$

Simplifying this,

$$B(p) = \frac{\theta^{c+2}}{p[\theta\Gamma(c+2)+\Gamma(c+4)]}\int_0^q x^{c+1}\left(1+\theta x^2\right)e^{-\theta x}dx,$$

and
$$L(p) = pB(p) = \frac{[\theta\gamma(c+2,\theta q)+\gamma(c+4,\theta q)}{\theta[\theta\Gamma(c+2)+\Gamma(c+4)]}.$$

10. Entropies

The entropy measures the amount of information present in a variable. It is the primary measure in information theory. The entropy of a random variable is the average level of “information” or “surprise” or “uncertainty” inherent in the variable’s possible outcomes.

10.1 Renyi entropy

It is defined as the index of diversity. Renyi means affable and even-tempered. It is mainly used in statistical physics, particularly in quantum information and equilibrium. For a given probability distribution, Renyi entropy is given by

$$\begin{aligned}
\theta(\beta) &= \frac{1}{1-\beta}\log\left(f^{\beta}(x)dx\right), \\
&= \frac{1}{1-\beta}\log\left[\left[\frac{\theta^{c+2}}{[\theta\Gamma(c+1)+\Gamma(c+3)]}\right]^{\beta}\int_0^{\infty} x^{\beta c}\left(1+\theta x^2\right)^{\beta} e^{-\beta\theta x}dx\right], \\
&= \frac{1}{1-\beta}\log\left[\left[\frac{\theta^{c+2}}{[\theta\Gamma(c+1)+\Gamma(c+3)]}\right]^{\beta}\int_0^{\infty} x^{\beta c}e^{-\beta\theta x}\left(1+\theta x^2\right)^{\beta} dx\right], \\
&= \frac{1}{1-\beta}\log\left[\left[\frac{\theta^{c+2}}{[\theta\Gamma(c+1)+\Gamma(c+3)]}\right]\sum_{j=0}^{\beta}\binom{\beta}{j}\left(\theta x^2\right)^{j}\int_0^{\infty} x^{\beta c}e^{-\beta\theta x}dx\right], \\
&= \frac{1}{1-\beta}\log\left[\left[\frac{\theta^{c+2}}{[\theta\Gamma(c+1)+\Gamma(c+3)]}\right]^{\beta}\sum_{j=0}^{\infty}\binom{\beta}{j}\frac{\Gamma(\beta c+2j+1)}{(\theta)^{\beta c+j+1}(\beta\theta)^{\beta c+2j+1}}\right].
\end{aligned}$$

10.2 Tsallis entropy

It is a generalization of Boltzmann-Gibbs (B-G) statistics. This entropy is non extensive. For a continuous random variable is defined as follows:

$$\begin{aligned}
S_{\lambda} &= \frac{1}{\lambda-1}\left[1-\int_0^{\infty} f^{\lambda}(x)dx\right], \\
&= \frac{1}{\lambda-1}\left\{1-\left[\frac{\theta^{c*2}}{[\theta\Gamma(c+1)+\Gamma(c+3)]}\right]^{\lambda}\int_0^{\infty} x^{\lambda c}e^{-\theta\lambda x}\left(1+\theta x^2\right)^{\lambda} dx\right\}, \\
&= \frac{1}{\lambda-1}\left\{1-\left[\frac{\theta^{c*2}}{[\theta\Gamma(c+1)+\Gamma(c+3)]}\right]^{\lambda}\sum_{j=0}^{\infty}\binom{\lambda}{j}\left(\theta x^2\right)^{j}\int_0^{\infty} x^{\lambda c}e^{-\theta\lambda x}dx\right\}, \\
&= \frac{1}{\lambda-1}\left\{1-\left[\frac{\theta^{c*2}}{[\theta\Gamma(c+1)+\Gamma(c+3)]}\right]^{\lambda}\sum_{j=0}^{\infty}\binom{\lambda}{j}\frac{\Gamma(\lambda c+2j+1)}{(\theta)^{\lambda c+j+1}(\lambda\theta)^{\lambda c+2j+1}}\right\}.
\end{aligned}$$

11. Data analysis

As we mentioned earlier, the weighted distribution is widely applied in various fields. Similarly, WCJD can also be applied in various research fields like medicine, biomedical science and engineering. Let us illustrate a dataset to show that WCJD can be better than one parameter CJD. Let us represent a data get, which represents the data of fossil CO_2 emissions per capita in India from 1981 to 2020. The data get is given as follows:

0.441829	0.446234	0.472227	0.473931	0.509576	0.534304
0.558213	0.589751	0.634558	0.664019	0.092245	0.722197
0.731148	0.757746	0.789656	0.839232	0.856934	0.858549
0.913936	0.923058	0.919146	0.930677	0.947477	0.989662
1.026231	1.073585	1.140592	1.211135	1.316904	1.351343
1.414156	1.539909	1.578007	1.673244	1.71655	1.780779
1.798018	1.899518	1.898949	1.750956	1.925088	

Table 1: Data by R. M. Andrew and G. P. Peters in DOI:10.5281/zenodo.72153.64

To compare the distributions, we study the criteria like Akaike Information Criterion (AIC), Akaike Information Criterion Corrected (AICC), Bayesian Information Criterion (BIC), and $-2\log L$. The better distribution is which compatible with lesser values of AIC, BIC, AICC, and $-2\log L$.

Distribution	MLE	S.E	-2 logL	AIC	BIC	AICC
CJD	$\tilde{\theta} = 2.0115891$	0.2129982	80.64461	82.64461	84.35818	82.74717
	$\tilde{\theta} = 5.609662$	1.050614	48.35771	52.35771	55.78485	52.67349
WCJD	$\hat{c} = 3.316918$	0.980331				

Table 2: MLE's, -2logL, AIC, BIC and AICC of the weighted Chris-Jerry distribution of data get

It can be obvious that the WCJD have the lesser AIC, BIC, AICC and $-2\log$ L values as analogize to CJD. Hence, we can finalize that the WCJD leads to better fit than the CJD.

12. Conclusion

In this study, we have presented a new extension of the Chris-Jerry distribution, known as the weighted Chris-Jerry distribution, with two parameters. The resiliency of the parameters in this distribution allows it to provide better results. The structural properties are well explained. Furthermore, the parameters are estimated using the maximum likelihood estimation method and tested using the likelihood ratio test. This distribution shows better compatibility than the Chris-Jerry distribution when applied to real lifetime datasets.

References

[1] D. R. Cox, *Some sampling problems in technology*, In: New Development in Survey Sampling, Johnson, N. L. and Smith, H., Jr (eds.) New York Wiley Interscjenge, **6(5)** (1969), 506–527.

[2] K. O. Chrjegoyus, J. O. Okechukwu, *Chris - Jerry distribution and its applications*, Asian J. Probab. Stat., **20(1)** (2022), 16–30.

[3] R. A. Fisher, *The effects of methods of ascertainment upon the estimate of frequencies*, Ann. Eugenics, **12(1)** (1934), 13–25.

[4] M. E Gbitany, B. Atieh, S. Nadarajah, *Lindley distribution and its application*, IMACS Ann. Comput. Appl. Math., **78(4)** (2008), 493–506.

[5] R. D. Gupta, D. Kundu, *A new class of weighted exponential distribution*, J. Theor. Appl. Stat., **43(6)** (2009), 621–634.

[6] L. G. B. Pinho, G. M. Cordeiro, J. C. Nobre, *The gamma exponentiated Weibull distribution*, J. Stat. Theor. Appl., **11(4)** (2012), 379–395.

[7] B. A. Para, T. R Jan, *On three parameter weighted Pareto type ii distribution; Properties and applications in medical science*, Appl. Math. Inf. Sci. Lett., **6(1)** (2018), 13–26.

[8] C. R. Rao, *On discrete distributions arising out of methods of ascertainment*, Sankhya - Series A, **27(2-4)** (1965), 311–324.

[9] A. A. Rather, C. Subramanian, *A new exponentiated distribution with engineering science applications*, J. Stat. Appl. Pro., **9(1)** (2020), 127–137.

[10] A. A. Rather, C. Subramanian, *Characterization and estimation of length-biaged weighted generalized uniform distribution*, IJSRMSS, **5(5)** (2018), 72–76.

[11] S. Sen, S. S. Maiti, N. Chandra, *The* X *gamma distribution: Statistical properties and application*, JMASM, **15(1)** (2016), 774–788.

[12] R. Shanker, K. K. Shukla, *On Poisson-weighted Lindley distribution and its applications*, JSR, **11(1)** (2019), 1–13.

[13] R. Shanker, K. K. Shukla, *Weighted Shanker distribution and its applications to model life time data*, JAQM, **12(2)** (2017), 245–265.

[14] P. C. Van Deusen, *Fitting assumed distributions to horizontal point sample diameters*, **32(1)** (1986), 146–148.

[15] M. Zelen, *Problems in cell kinetic and the early detection of disease*, In: Reliability and Biometry, F. Proschan& R. J. Sering, eds, SIAM, Philadelphia, **6(5)** (1974), 701–706.

(received 20.09.2023; in revised form 07.04.2025; available online 25.10.2025)

Department of Statistics, Annamalai University, Annamalai Nagar, Tamil Nadu, India
E-mail: manistat@yahoo.co.in
ORCID iD: https://orcid.org/0000-0002-9625-0658

Department of Statistics, Annamalai University, Annamalai Nagar, Tamil Nadu, India
E-mail: subhashreem17@gmail.com
ORCID iD: https://orcid.org/0009-0003-8263-1622

Symbiosis Statistical Institute, Symbiosis International (Deemed University), Pune-411004, India
E-mail: aafaq7741@gmail.com
ORCID iD: https://orcid.org/0000-0001-8962-1660

Symbiosis Institute of Technology, Symbiosis International (Deemed) University, Pune-412115, India
E-mail: shahidwani177@gmail.com
ORCID iD: https://orcid.org/0000-0001-6484-469X